

Dynamic Asset Pricing with Passive Investing

RUGGERO JAPPELLI*

[Latest version here](#)

January 6, 2024

ABSTRACT

The paper presents and tests a tractable capital asset pricing model with active investors, whose portfolio choice responds to news, and constrained investors, who hold a static allocation across asset classes even if risk and return change over time. The model shows that i) equity investments resulting from static allocation strategies provide a floor to the value of corporate equities, which trade above their discounted future dividends; ii) allocation strategies generate price volatility in excess of, but connected to, the volatility of earnings; iii) the precision of the information that stock prices convey about future earnings decreases when expected returns are low and volatile. These predictions are confirmed in the aggregate time series and cross section of stocks.

JEL classification: G11, G12, G14, G23.

Keywords: Asset pricing, asset class effect, investment mandates, price efficiency.

*Leibniz Institute for Financial Research SAFE and Goethe University Frankfurt, House of Finance, 60323, Germany, jappelli@safe-frankfurt.de.

I thank my supervisors, Florian Heider, Lorian Pelizzon, Alberto Plazzi, and Marti Subrahmanyam, for their continued guidance. I also thank Klaus Adam, Rui Albuquerque, Patrick Augustin, Andrea Beltratti, Giovanni Cocco (discussant), Wenxin Du, Bernard Dumas (discussant), Francesco Franzoni, Xavier Gabaix, Fabio Girardi, Alexander Hillert, Evan Jo, Holger Kraft, Carmelo Latino, Martin Lettau, Lorian Mancini, Andrea Modena, Marco Pagano, Max Riedel, Daniel Schmidt, Christian Schlag, Stephan Siegel, Quentin Vandeweyer, and conference and seminar participants at the Bonn Frankfurt Mannheim Workshop, Goethe University Frankfurt, the Inter-Finance doctoral seminar, the Leibniz Institute for Financial Research SAFE, the Finance Ph.D. Final Countdown organized by the Nova School of Business and Economics, the University of Mannheim, USI Lugano, and the Venice Finance Workshop for their valuable comments, NYU Stern for the hospitality during the initial stage of this research, and the Leibniz Institute for Financial Research SAFE for generous research support. This work was awarded as the best Ph.D. paper at the 6th Asset Pricing Conference hosted by the Long-Term Investors at Collegio Carlo Alberto. Errors are the author's responsibility.

1 Introduction

One of the most interesting developments in financial markets over the last three decades is the rising importance of professional asset management.¹ Asset managers often operate under mandates to follow static portfolio allocation rules that limit their ability to exploit investment opportunities. For example, most mutual funds (MFs) specify constant targets for the proportion of their wealth to be invested across asset classes, and exchange-traded funds (ETFs) often replicate the performance of an index. In contrast to this stylized fact, standard approaches to asset pricing consider active agents who continuously optimize their investment choices. For example, in the intertemporal capital asset pricing model (ICAPM) the portfolio allocation decisions between safe and risky assets reflect stochastic changes in the investment opportunity set. This paper presents and tests a tractable model of the stock market where active investors interact with investors with static portfolio rules against the backdrop of exogenous earnings in the economy.

The rising importance of professional investors who operate under static portfolio rules poses a natural question. What ties the dynamics of stock prices to fundamentals, in financial markets where the allocation of capital across asset classes is gradually less attached to news announcements? This paper contributes to a program of research on noise in financial markets by examining the price pressure that results from static allocation mandates, which are unrelated to news. It is the first attempt to price stocks in relation to the level of wealth of institutions using static asset allocation rules.

The paper offers three results. The first suggests that allocation mandates generate an “*asset class effect*,” whereby the value of corporate equities is at least as high as the wealth allocated to the equity asset class by static allocation strategies – Federal Reserve data value U.S. equities at \$65 Tn, of which \$17 Tn are held by MFs and ETFs. These investors keep a static proportion of their wealth allocated to equities, even if the risk and return profile of the equity asset class changes dynamically. The aggregate influence of asset allocation strategies thus generates cash-in-the-market pricing features. Second, allocation strategies convey information about investors’ response to news. Market movements that impact the wealth of investors with static allocations prompt reinvestments in fixed proportions. Thus, allocation strategies give rise to procyclical price pressure and generate conditional price volatility that exceeds the volatility of fundamentals but is still connected to it. Third, the paper uncovers a relationship between the degree to which stocks are informative about future earnings and their expected risk and return.

¹Grossman (1995), Stein (2009), and Stambaugh (2014) discuss the asset pricing effects of professional asset management.

This paper proposes an ICAPM with heterogeneous agents to examine the effects of intertemporal changes in the investment opportunities. Active investors incorporate the information on prospective fundamentals in asset prices, but when market risk increases, these investors reduce their exposure to stocks. In contrast, investors with asset allocation mandates maintain their portfolios unchanged even when markets are volatile, creating with their demand a floor to stock prices. The scope for price deviations from fundamentals is thus larger in volatile markets than it is when markets are stable, all else equal, underscoring a broader connection between price informativeness and the market conditions. In general, as the importance of passive investments rises, the information content of stock prices drops. Ultimately, for passive investors to hold more stocks, active investors must have the incentive to hold less stocks. In equilibrium, the incentive of active investors, in the form of the expected return of the position compared to its risk, is thus connected to the information content of stock prices.

The model works as follows. There are two asset classes – one riskless and the other with risky dividends – and two groups of agents, which differ in their investing styles. Active investors, such as sophisticated households and hedge funds (HFs), optimally revise their portfolio choice upon the arrival of information to the market. On the other hand, passive investors such as MFs and ETFs are defined in *broad* terms as investors who do not optimize their portfolio intertemporally, but rather hold a constant allocation between the two asset classes regardless of risk and return considerations. These investors are passive over *time*, as they do not adjust their portfolio allocation to reflect news and thus to changes in the investment opportunity set. Passive investors receive stochastic wealth flows, which cannot be replicated using traded assets and thus generate market incompleteness.² Financial markets feature rational expectations, symmetric information, and are free of arbitrage, so that the pricing kernel uniquely results from the preferences of active investors. Given the absence of management fees and the impossibility of outperforming the market consistently, the wealth of both active and passive investors grows at the same expected risk-adjusted rate, and neither of the two groups of agents dies out over time. Active investors do not arbitrage away the effect of passive investors on stock prices, because the price pressure generated by passive investments is persistent. Passive investments unrelated to news increase the price of the risky asset class, reduce its expected return, and increase its price volatility, thus crowding out active investments. The structure outlined can be employed to discuss the effects of asset allocation mandates in relation to two aspects: i) the aggregate stock market; and ii) the cross section of stocks.

²Capital flows to institutional funds influence their trades, affecting asset prices via this channel (Coval and Stafford, 2007).

In the model, the price of a stock differs from the present discounted value of its dividends. Standard arguments suggest that if a stock is priced above its dividend stream, active investors would sell the stock short to replicate its dividend stream and profit from the difference until correcting the mispricing. This logic does not carry through in the presence of passive investors, for a simple reason. Static portfolio weights decouple the demand of passive investors from risk and return considerations, generating price pressure. Mandates are permanent commitments, and produce a level of price pressure which reliably forecasts the future levels of price pressure. Moreover, stocks have an infinite maturity and lack a terminal condition for valuation. Passive investing rules thus change the qualitative nature of the optimization problem of the active investors, who face demand pressure in the present and expect demand pressure going forward. Even if a stock trades above its discounted dividend stream, no rational agent would push its price below the expected price in the future.

Wealth statically allocated to the stock market generates a rational bubble, whereby persistent price deviations from economic fundamentals do not constitute arbitrage opportunities. Portfolio holdings are common knowledge and guarantee price uniqueness. In the derived equilibrium with active and passive investing styles, the workings of active investors ensure the equivalence between the price of the risky asset and the discounted value of two terms, the dividend stream and the wealth passively allocated to the stock market. Earnings are exogenous, but the wealth dynamics of passive investors result *endogenously* from market movements, leading to the reinvestment of capital gains and to the resulting wealth amplification effects. The dynamic structure of this heterogeneous investors framework is advantageous, since the magnitude of the effects derived in equilibrium varies over time with the proportion of shares outstanding held by investors with asset allocation mandates. For instance, the secular trend toward passive investing gradually strengthens wealth amplification effects, in line with the data discussed below.

To my knowledge, this paper is the first to develop the notion of “*asset class effect*,” whereby the wealth invested in stocks under static allocation strategies provides a floor to the aggregate value of corporate equities. A natural concern is that the wealth invested under static allocation strategies may partly reflect information, and needs to be regarded as endogenous to the economy. For example, retail investors could liquidate part of their fund shares during market turmoils. To incorporate this consideration, the model is extended to consider the positive association between capital flows into MFs and ETFs and the current and past performance of the economy, an empirically relevant feature that strengthens the baseline results and generates persistence in aggregate returns.

The model presented thus far with two agents, active and passive investors, provides theoretical predictions for the time series of aggregate stock prices. However, a meaningful characterization of the *cross section* of stocks requires a minimum of three groups of agents. Passive investors such as asset allocation MFs and index-tracking ETFs have constant allocation rules across asset classes, yet they diverge in their investment approaches in the cross section. Index-tracking ETFs have fixed portfolio weights even within the equity asset class. By contrast, asset allocation MFs optimally select the stocks in their portfolio, but they still passively invest on the stock market a fixed proportion of wealth regardless of changes in the opportunity set. Active investors such as HFs are the least constrained agents, and allocate a proportion of their wealth based on stochastic changes in the opportunity set to an optimal mix of stocks. The analysis of the cross section serves two purposes. First, it underscores that an asset class effect tied to allocation mandates arises over and above the known index inclusion effect. In analogy to stocks included into a benchmark index that are overpriced relative to their discounted dividend stream, certain securities are overpriced relative to their discounted cash flows in association with their inclusion in the equity *asset class*, targeted by the allocation mandate of professional investors.³ In terms of price dynamics, passive allocation mandates affect conditional expected returns, volatility, and pairwise correlations between stocks. Second, the cross section of stocks facilitates the empirical identification strategy.

The main predictions of the model can be summarized as follows. First, given earnings and discount rates, the price of stocks rises with the wealth invested under the mandate of allocation to equities. Second, static investing strategies amplify the responsiveness of stock returns to earnings surprises. Third, price informativeness rises with the incentive of the active investors to hold stocks, namely their risk/return trade-off, and decreases with the position of passive investors.⁴ The paper presents empirical evidence consistent with the predictions outlined, which are confirmed by the examination of market and holdings data along two dimensions: i) the time series of the aggregate stock market; and ii) the cross section of stocks. Aggregate time series patterns suggest that the proportion of the stock market held by MFs and ETFs correlates positively with equity valuation ratios. In the model, this effect achieves since passive investments influence prices but earnings are exogenous. This paper may thus help in explaining the structural breaks in the price/earnings multiple documented by [Lettau, Ludvigson, and Wachter \(2007\)](#), as well as in the value of the U.S. market capitalization relative to its gross domestic

³The asset class effect can be identified by exploiting the variation over time of the wealth allocated to the stock market.

⁴Other testable results developed in this paper are left for future research.

product. Dynamically, the model suggests that the sensitivity of aggregate stock prices to news correlates positively with the stock market ownership share of MFs and ETFs. Empirically, this first attempt to enrich time-series models of market returns using holdings data suggests that the same news should have a larger effect on returns depending on the *ownership structure* of the stock market. This prediction is strongly confirmed by a parsimonious component model for conditional volatility that blends daily returns and quarterly holdings data.

The cross section of stocks offers a clean laboratory for identification. To assess whether the ownership share of passive investors is associated with a higher sensitivity of stock returns to earnings via the proposed wealth channel, the paper relies on an event study of the abnormal returns to corporate earnings announcements. In a comprehensive sample of more than 5,000 U.S. stocks with daily data ranging from 1998 to 2018, the effect of standardized unexpected earnings on abnormal returns is significantly amplified by the wealth passively tracking the stock. The baseline earnings response coefficient of 0.276 increases to 0.380 at the median of the distribution of passive investments, with estimates that are robust to alternative statistical models for normal returns and perturbations of the event window.

In terms of forecasting price efficiency, a study around the yearly Russell 1000/2000 reconstitution cutoff confirms that stocks prices that are locally causally associated with exogenous demand pressure have significantly lower forecasting price efficiency for the future earnings of the company. With respect to their economic interpretation, the empirical patterns documented are in agreement with each other. By its nature, the accounting system recognizes information with a lag with respect to the stock market. Hence, when the stock price is less informative about future earnings, its responsiveness to the release of earnings reports tends to be higher. The result that firms that attract higher passive investments respond more strongly to earnings announcements is thus entirely consistent with their stock prices being less informative about future earnings. Overall, the empirical evidence is in agreement with the predictions of the model.

The paper is organized as follows. Section 2 places the paper against the backdrop of the literature. Section 3 presents an intertemporal model of asset price dynamics in financial markets where active and passive investors continuously trade with each other on the arrival of news about fundamentals and wealth flows. Section 4 discusses implications for the cross section of stocks. Section 5 presents several extensions of the model. Section 6 then tests the main predictions. Section 7 offers concluding remarks. The Appendix contains derivations, proofs, and additional results.

2 The Literature

The contribution of this paper is to develop a tractable equilibrium model where the wealth of investors with static asset allocation strategies affect asset prices which generates new predictions supported by the data. The paper thus relates to foundational contributions on the effects of demand forces on asset prices offered by [Scholes \(1972\)](#), [Harris and Gurel \(1986\)](#), and [Shleifer \(1986\)](#). These effects are well documented in the literature. For example, the inclusion of stocks into a popular index used by asset managers to benchmark their performance generates abnormal returns and results into an excess comovement of the index constituents relative to their fundamentals ([Greenwood, 2008](#)). [Pavlova and Sikorskaya \(2023\)](#) estimate that the effects of index rebalancing on stock returns persists for up to 5 years. In comparison with the literature on index inclusion effects, which studies the relative price of stocks in the cross section, this paper focuses on passive investing rules at a more general level than index investments, namely asset allocation mandates between stocks and risk-free investments. In doing so, this paper considers the stock market at a more aggregate scale and develops the notion of *asset class effect*, whereby the entire set of corporate equities is valued at least as the wealth allocated to equity by mandate. The asset class effect, the main result of this paper, is important even for the pricing of stocks not included into any benchmark.

Several papers in the recent literature study the effects of professional asset management on market prices. In the intermediary asset pricing literature developed by [He and Krishnamurthy \(2013\)](#) and [Brunnermeier and Sannikov \(2014\)](#), active specialized intermediaries affect the valuation of securities. This literature examines the effects of the leverage decisions of optimizing intermediaries on asset prices, while this paper focuses on investment funds with constant asset allocation, and thus leverage.⁵ Relatedly, [Vayanos and Woolley \(2013\)](#) develop a model where stock prices reflect the management costs and the flows of wealth into mutual funds, and show that the relationship between past returns and flows across funds induces momentum and reversal dynamics. [Basak and Pavlova \(2013\)](#) show that the effort of asset managers to outperform their benchmark generates procyclical leverage decisions that amplify the level and the volatility of the index.⁶ But the focus on benchmark indexes that characterizes these contributions does not permit them to address the effects of the wealth passively invested in financial

⁵Even if the leverage of optimizing intermediaries matters for asset pricing ([Haddad and Muir, 2021](#)), this paper deliberately uses the simplest possible specification for active investors so as to examine the asset pricing effects of passive investments.

⁶Appendix D elaborates in more detail on the relationship with [Basak and Pavlova \(2013\)](#), which this paper generalizes to investment mandates of asset allocation across asset classes as well as to stock prices endogenous to the wealth dynamics.

markets through mandates of asset allocation *across* asset classes. This paper focuses on passive trading rules defined in general terms as fixed allocation rules, both between and within asset class, and is a first attempt to price the stock market in relation to the level of wealth allocated to it by static investment mandates. While traditional benchmarking considerations generate cross-sectional index inclusion effects, this paper primarily studies investors that are passive over *time* and set static portfolio shares across asset classes, thus engendering the asset class effect. Moreover, this paper documents that asset allocation mandates result into price volatility in excess of, but still connected to, the volatility of fundamentals, and uncovers a connection between the information content of stock prices and their risk and return.⁷

Furthermore, this paper relates to the research on the effects of investment mandates on capital markets. [Kojien and Yogo \(2019\)](#) show that the heterogeneity in demand for the characteristics of assets affects their going market prices. Other papers related to the effects of demand on asset valuations include [Pandolfi and Williams \(2019\)](#), [Kojien, Richmond, and Yogo \(2020\)](#), [Coppola \(2021\)](#), and [Haddad, Huebner, and Loualiche \(2021\)](#). The interest in the relationship between fundamentals and asset price dynamics underscores the contribution of the present paper to this literature. This paper also relates to [Gabaix and Kojien \(2022\)](#), who document that the asset allocation constraints of institutional investors amplify the effects of flows on asset prices. In the latter paper, however, the response of prices to earnings news is attenuated by the presence of investors with mandates, while this paper shows theoretically and empirically that mandates result in price volatility in excess of the volatility of fundamentals, but still connected to it. This paper differs in several other ways. Stock prices here reflect dividends and the level of wealth of passive investors, rather than dividends and stochastic flows. Wealth differs from stochastic investment flows since it grows over time, and it does so with the market itself, leading to different economic mechanisms and results, as detailed in Appendix D. Differently from previous studies, this is a paper where investors with mandates interact with optimizing agents, who can accommodate an unlimited amount of temporary noisy investment flows. Such optimizing agents would not, however, take the opposite side of investments committed to the asset class by a mandate. For example, active investors in the model would arbitrage away trades arising for temporary motives, but would not bet on a reversal after the purchases of ETFs by the Bank of Japan, which have persistent price effect ([Barbon and Gianinazzi, 2019](#)). In sum, this paper proposes a new unified setup for the time series, the cross section, and the information content of stock prices.

⁷[Coles, Heath, and Ringgenberg \(2022\)](#) and [Gârleanu and Pedersen \(2022\)](#) focus on passive investing and market efficiency.

In broader terms, this paper is part of an agenda on investors' heterogeneity in financial markets. This area of research includes studies on the effects of quantitative easing on fixed income securities, focus of [Vayanos and Vila \(2021\)](#), [Gourinchas, Ray, and Vayanos \(2022\)](#), and [Jappelli, Pelizzon, and Subrahmanyam \(2023\)](#), as well as on the influence of passive investing style on the commodities markets studied by [Tang and Xiong \(2012\)](#).

3 The Model

The model is cast in continuous time over an infinite horizon and considers financial markets with two groups of economic agents, whose portfolio choice is either unrestricted or constrained by a mandate of asset allocation.

3.1 Assets

There are two classes of assets, one is risky and the other one is riskless. Suppose the securities market operates continuously, and let P_t denote the ex-dividend share price of the risky asset at time t . Real earnings accruing to the risky asset are denoted by E_t and follow a stochastic differential equation with drift m and diffusion ω ,

$$dE_t = mdt + \omega dB_t, \tag{1}$$

where B_t is a Brownian motion which generates the filtration $\{\mathcal{F}_t\}$. The arithmetic process is a rich specification, as the model is cast in real terms and real earnings grow linearly, as will be illustrated in [Figure 2](#). An elastically supplied riskless asset yields the instantaneous real rate of return r . There are no arbitrage opportunities, and agents form expectations rationally. Throughout, let \mathbb{Q} denote the risk-neutral measure, and define μ_t and σ_t as the instantaneous price drift and diffusion, respectively. For simplicity, the risky asset has a constant payout ratio, so that the dividend per share D_t is a fixed proportion a of the earnings.⁸ The paper departs from the previous literature by grouping market participants into two categories: active and passive investors. Active investors are optimizing agents who incorporate news about economic fundamentals in asset prices. The wealth of active investors is denoted by W_t . Conversely, passive investors use a fixed portfolio allocation rule that reacts to assets under management and market prices, but does not react to the arrival of new information. The wealth of passive investors is V_t . The

⁸Rights issues influencing the value of shares without affecting their supply to the public can account for negative dividends.

total demand for stocks is composed of actively and passively invested funds. Agents face common price and information \mathcal{F}_t . In brief, active investors set their holdings optimally given the information available on the market, while passive investors follow their asset allocation mandate without regard to the revelation of information. The paper details the behavior of the two groups of investors and derives the resulting equilibrium. Where possible, the time index is omitted to ease equation presentation.⁹

3.2 Active Investing

Active investors can be thought of as households, broker dealers, and HFs, who optimize CARA preferences $U(c_t) = -e^{-\delta t - \gamma c}$, where c denotes consumption, and δ and γ are respectively patience and risk aversion parameters. These participants (Merton (1973) style investors) respond to news about earnings, and their portfolio choices change continuously over time with the arrival of new public information.¹⁰ Active investors incorporate news about economic fundamentals into asset prices. They control their consumption and investment policies so as to maximize their expected intertemporal utility over an infinite time horizon, while respecting their budget constraint and transversality condition.¹¹ Their indirect utility function is

$$J \equiv \max_{\{c, X\}} \mathbb{E}_t \left[\int_t^\infty U(c_s) ds \right],$$

$$\text{s.t. } dW = (rW - c)dt + XdY, \quad \lim_{h \rightarrow \infty} \mathbb{E} \left[J_{t+h} \right] = 0. \quad (2)$$

In the above, $dY = (D - rP)dt + dP$ is the return of a share of the risky asset financed at the risk-free rate. This classical problem is similar to that discussed by Veronesi (1999) and others, with the important distinction that in the present paper active investors are aware of the presence of other participants in the market whose demand is unrelated to fundamentals, as detailed below.

⁹To improve readability, only the equations to which there is a subsequent reference are numbered.

¹⁰Chien, Cole, and Lustig (2011, 2012) examine the effects of intermittent portfolio rebalancing for the price of assets.

¹¹Hodor and Zapatero (2023) study the effects of investors' trading horizons on the term structure of equity.

3.3 Passive Investing

The paper explores the consequences of agents who are *not* optimizing for the dynamics of asset prices. Passive investors are defined as investors who hold arbitrary fixed allocation shares regardless of the going market prices, and can be described by means of mechanical portfolio allocation rules. As documented by [Gabaix and Koijen \(2022\)](#), a large class of investors holds approximately constant equity shares.¹²

Consider the demand for shares of a professional fund with the mandate to hold the equity share θ . For example, professional asset managers frequently anchor their portfolios to a 60/40 equity/bond portfolio structure. To fix ideas, [Figure A.1](#) shows that MFs and ETFs have average θ values of 0.93 and 0.96, respectively. Throughout, without loss of generality the individual asset allocation funds are aggregated into a representative fund for simplicity of exposure.¹³ As a group, passive investors wish to have a constant proportion of their wealth invested in the stock market. Thus, passive investors demand a number of shares of the risky asset class Q_t equal to

$$Q_t = \theta V_t / P_t. \quad (3)$$

This formulation implies that the fund has a downward sloping demand for the risky asset, in the spirit of early contributions by [Harris and Gurel \(1986\)](#) and [Shleifer \(1986\)](#). The representative passive fund balances its portfolio continuously, just as active investors do.¹⁴ The representative fund with θ less than unity is thus contrarian. For example, if the stock price decreases, the risky asset becomes less expensive to hold and the passive fund purchases more shares of the risky asset, regardless of the determinants of the price change. The passive fund does not directly adjust its portfolio shares to react to news about fundamentals, as instead active investors do.¹⁵ The passive fund reinvests the dividends generated by the risky asset, and receives wealth inflows and redemption

¹²This fixed portfolio composition is illustrated in [Figure A.1](#), which replicates [Figure 1](#) in [Gabaix and Koijen \(2022\)](#) along with a time series of the realized Sharpe ratio of the equity asset class. Interestingly, the variation of the equity share of MFs and ETFs around their average, respectively 0.98 and 0.93, is minimal. Nonetheless, over time the Sharpe ratio reaches a peak of 1.75 and a trough of 0.25, reflecting sizeable and stochastic changes in the investment opportunity set. For example, during the global financial crisis, investors with asset allocation mandates did not fly to the safe asset class.

¹³For example, fund x managing $V_x = 100\$$ with equity allocation $\theta_x = 0.5$ and fund y operating $V_y = 200\$$ with $\theta_y = 0.75$ aggregate into a representative fund with wealth $V = 300\$$ which invests into stocks the average of the mandates of the two funds weighted on their wealth, $\theta = 0.67$. The subsequent wealth flows into each of the funds are scaled equivalently.

¹⁴We should, if anything, expect passive investors to rebalance more regularly than active investors. ETFs, for instance, are committed by their mandates to rebalance their portfolios daily so as to replicate their benchmark index.

¹⁵[Buffa, Vayanos, and Woolley \(2022\)](#) study the effects of deviations from the mandates constrained by agency frictions.

requests. The law of motion of passively invested wealth is thus

$$dV = V[(1 - \theta)r dt + \theta(dP + Ddt)/P] + \pi dF \quad (4)$$

with F being a Brownian motion under the risk-neutral measure adapted to $\{\mathcal{F}_t\}$, which describes the flows of wealth to the assets under management of the fund net of share redemption requests, and π being a loading parameter.¹⁶ The process F summarizes the decisions of economic agents to invest in or divest from the passive fund. For the time being, suppose that wealth flows to the passive fund are uncorrelated with the economic fundamentals.¹⁷ Equation (4) is a special case of Equation (2) when portfolio shares are constrained by the asset allocation mandate θ , and the fund's wealth is subject to flow risk πdF .¹⁸ This formulation shows that the wealth passively invested on the stock market is, on average, higher following high realizations of stock returns dP/P in the recent past, since returns are cumulative. As opposed to the forward-looking nature of actively invested wealth, wealth invested using the passive style has memory of the past.¹⁹ This feature will be shown to lead to interesting predictability patterns.

3.4 Market Clearing

Assume a fixed supply of shares \bar{S} , normalized to unity without any loss of generality. The fraction of shares held by active investors is denoted by X_t , and the fraction of shares held by passive investors by Q_t . The market clearing condition is

$$X_t(P, D, V) + Q_t(P, V) = \bar{S}. \quad (5)$$

3.5 Interpretation

Figure 1 shows the ownership structure of the U.S. equity market over time, highlighting that the importance of delegated portfolios is on the rise, particularly taking off around the '90s. By the end of the sample, households only directly held around 40% of U.S. equity markets. Meanwhile, MFs and ETFs combined ownership shares approximately accounted for 35% of the total market value, with the remaining proportion of the stock

¹⁶Equivalently, $dV = rV(1 - \theta)dt + Q(dP + Ddt) + \pi dF$.

¹⁷The results in Section 5 are derived using a more general process where flows are correlated with the past performance.

¹⁸As a result of large negative redemption flows F , with low probability the process V could become negative while funds have non-negative wealth. The dynamics near the barrier could be regulated by adding a term ηdL , where η is a speed of reflection parameter and L is the local time of V at zero. This correction would not affect the content of the results.

¹⁹Figure A.2 illustrates this property using a binomial tree.

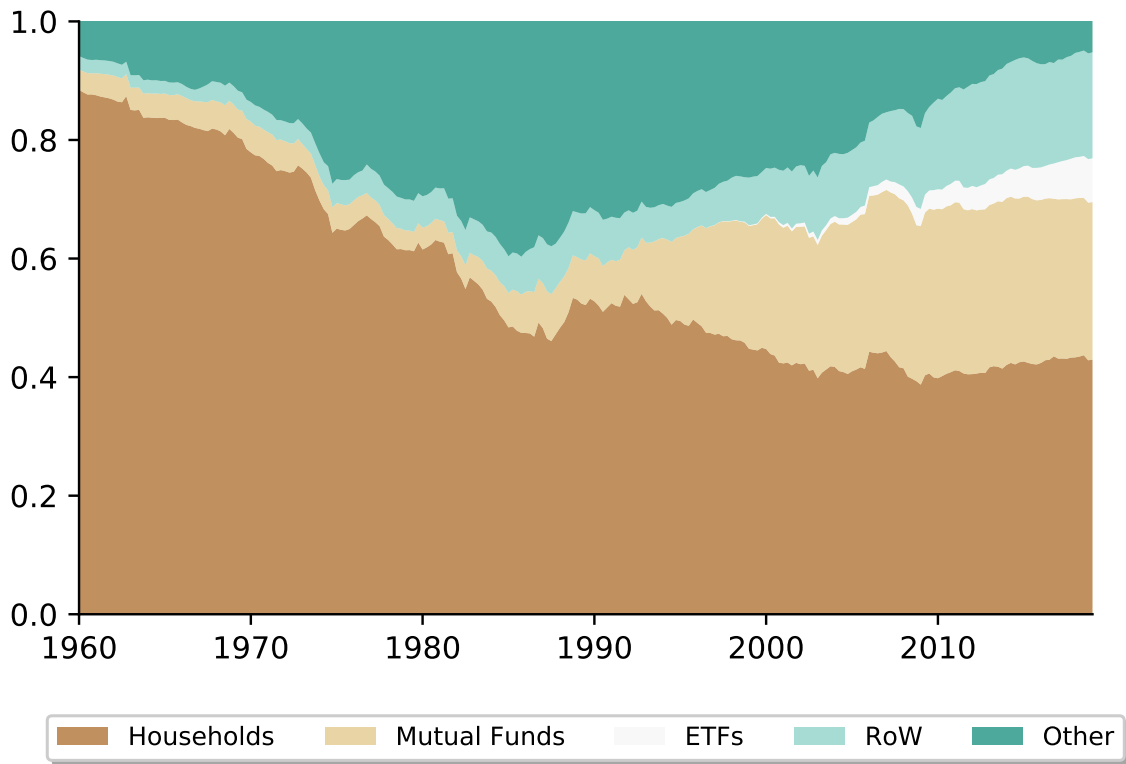


FIGURE 1: **U.S. Equity Ownership.** The figure shows the composition of investors in U.S. corporate equities using Financial Account data from the Fed. RoW denotes rest of the world.

market mostly held by foreign investors.

This paper considers the presence of investors with allocation mandates as a given feature of the financial markets, and examines the equilibrium investment policy of intertemporally optimizing investors such as sophisticated households and HFs. To parsimoniously model the heterogeneous composition of investors, it is assumed that active households trade on their own account or through the formation of HFs so as to optimize their utility. Standard portfolio theory described above applies to these active investors. By contrast, other households are less attentive to the stock market and delegate their investment decisions to professional portfolio managers, such as MFs and ETFs. When investing the wealth of these households, professional managers operate under asset allocation mandates, which could be motivated by unmodeled agency considerations along the lines of [He and Xiong \(2013\)](#). Asset allocation mandates present an intriguing aspect of capital markets, by constraining a large set of investors from exploiting intertemporal changes in the investment opportunity set.

3.6 Equilibrium

Definition. *The equilibrium consists of a price P of the risky asset such that the supply \bar{S} of shares is equal to their demand $X + Q$. The investment decisions of active investors maximize their intertemporal utility over consumption c , given the level of their wealth, the earnings of the portfolio constituents, and the price of the risky asset. Given the flows of funds, passive investors allocate a fixed share θ of their wealth in the stock market. Thus, the stock market is described by three conditions.*

$$X + Q = \bar{S}, \quad 0 = \max_{\{c, X\}} U(c) + \frac{\mathbb{E}_t[dJ(W, V)]}{dt}, \quad Q = \theta V/P.$$

The first equation is the market clearing condition, the second is the Bellman equation of the active investors, and the third one the asset allocation mandate of passive investors. The equilibrium concept is Walrasian, whereby trades follow the determination of the price which clears demand and supply and every share trades at the going market price.

3.7 Equilibrium without Passive Investing

It is useful to derive a benchmark equilibrium in the absence of passive investors, where the effect of their price pressure is muted. This can be achieved by the restriction $\theta = 0$, eliminating the stock market participation of passive investors.

Proposition 1. *In the absence of passive investors, the price of the risky asset equals the present discounted value of dividends.*

$$P_t = \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} D_s ds \right]}_{\text{Fundamentals}}.$$

Moreover, for suitable time preference parameter β defined in the Appendix,

$$X_t(P, D) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2},$$

$$c_t(W) = rW_t + \frac{1}{\gamma}(\beta - \log r).$$

Proof. Special case of Proposition 2. See also Veronesi (1999).

By replacing the earnings process in Proposition 1, the standard Merton result obtains.

$$P_t = p_g + p_D D_t + p_m m, \quad (6)$$

where the parameters governing the price level are $p_g = -\frac{\gamma\omega^2}{r^2}$, $p_D = \frac{1}{r}$, and $p_m = \frac{1}{r^2}$.²⁰ In this standard equilibrium, where the only uncertainty is about earnings, $\sigma = \omega$ and volatility does not vary over time. Moreover, the expected price changes $\mu = \frac{m}{r}$ is constant and the price on earnings multiple $\frac{P}{E}$ features limited time variation. Finally, the market is entirely held by agents who actively respond to news about economic fundamentals. These features appear in stark contrast with the empirical facts documented in the literature. Example 1 clarifies the price formation process. It appears trivial but serves as a useful benchmark. Recall that in this class of equilibrium models, each share trades at the same market price.

Example 1. *Suppose that the risky asset trades at $P = \$100$ and is entirely held by active investors, thus $X = 1$. As firms announce good earnings, the present discounted value of future dividends increases by $\$20$. In the equilibrium, active investors want to hold more shares, and following their heightened demand, the price endogenously increases to $P + dP = \$120$. Active investors realize capital gains $dW = \$20$ and increase their consumption level.*

Proposition 1 thus illustrates a traditional Gordon equity valuation model. This pricing equation is usually obtained recursively from the definition of stock returns, in the absence of so-called “rational bubbles.” On the other hand, any additional term v_t which satisfies $v_t = \mathbb{E}_t^{\mathbb{Q}}[e^{-r dt} v_{t+dt}]$ can appear on both sides of Equation (6) and be consistent with rational price deviations from economic fundamentals. [Tirole \(1982\)](#) argues that positive price bubbles are incompatible with rationality, since active agents could otherwise sell the asset short and replicate its dividend stream. This logic fails to capture that a large class of market participants is composed of passive investors, who follow a mechanical portfolio rule. Passive investments are publicly known and, since there is no arbitrage opportunity, are expected to grow at the same rate as that of the market in risk-adjusted terms. This is the requisite conditions for persistent effects on asset prices in rational markets. Equivalently, the absence of passive investors is necessary for the described equivalence between a stock price and the present discounted value of its dividends. Far from a theoretical case, passive capital allocation is becoming the standard

²⁰For ease of notation, the dividend payout ratio a is assumed equal to 1, but the general case simply achieves by multiplying the parameters p_g , p_D and p_m for the constant dividend payout ratio a .

investment style. Importantly, passive investors do not give rise to sunspots, since their wealth is known to market watchers and the equilibrium detailed in the following result is thus unique.

3.8 Equilibrium with Active and Passive Investing

When both active and passive investors trade, prices deviate from economic fundamentals.

Proposition 2. *In the presence of passive investors, the price of the risky asset is the sum of the present discounted value of the future stream of dividends and of the future passive investments in the risky asset class.*

$$P_t = \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} D_s ds \right]}_{\text{Fundamentals}} + \theta \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} V_{t+dt} \right]}_{\text{Wealth Allocated}}.$$

Equity shares and consumption maximizing the Hamilton-Jacobi-Bellman Equation are

$$X_t(P, D, V) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma} Q_t,$$

$$c_t(W, V) = rW_t + \frac{1}{\gamma} (g(V_t) + \beta - \log r).$$

Proof. See Appendix A.

In the above, $g(V_t)$ enables active investors to hedge the effect of passive investments on their investment opportunity set.²¹ It is commonly believed in financial economics that asset prices should equal the present value of expected discounted cash flows. By contrast, Proposition 2 shows that, above and beyond the variation in the discount rates of the marginal agents and the cash flows of the security, the market price interacts with the current and expected demand of nonfundamental investors, in the spirit of a recent strand of the literature (Gabaix and Koijen, 2022).²² This paper is however the first to make the point that the stock price is influenced by the *stock* of wealth of the passive investors, rather than stochastic wealth *flows*. From the wealth dynamics of passive investors in Equation (4), flows are part of the wealth of passive investors, which more generally also responds to the rate of return on the risk-free and risky assets.

²¹Hedging terms associated with the investment of other agents also appear in Kraft, Meyer-Wehmann, and Seifried (2020), who examine the asset pricing effects of active investors with relative wealth concerns.

²²Relatedly, Brunnermeier, Merkel, and Sannikov (2022) show that the price of safe assets deviate from their expected discounted future cash flows as a result of the service flows deriving from their negative exposure to aggregate shocks.

The traditional stochastic discount factor approach can comfortably be applied in the context of this model. However, the derived equilibrium illustrates that prices may well change even if the discount rate of active investors or the fundamentals have not. The stock price in Proposition 2 can be equivalently expressed in closed form as

$$P_t = \text{PDV}_t(D) + \theta V_t, \quad (7)$$

where $\text{PDV}_t(D) = p_\gamma + p_D D_t + p_m m$ stands for the present discounted value of dividends and incorporates risk corrections. Moreover, $p_\gamma = -\frac{1}{r} \left(\frac{\omega}{r} + \theta \pi \right)^2$ is the discount required by the active investors as a compensation for the exposure of prices to fundamental and flow risk, and is thus larger in presence of passive investors.

The equilibrium price in Equation (7) has the strong Markov property. The effect of passive investments on market prices is equal to passive wealth multiplied by the asset allocation mandate, θV . The price effect of passive investments is, therefore, tied to its magnitude and persistence. This feature is due to the workings of arbitrage, which directly imply that passively invested wealth θV_t grows at the risk-free rate r under the risk-neutral measure. Put differently, in the absence of commissions and fees, the expected returns from the active and passive investing styles are equivalent, in the spirit of [Sharpe \(1991\)](#). The passive investing style generates the persistent effects of a “rational bubble.” The solution of the model shows that changes in the wealth invested on the stock market in a manner unrelated to its prospective fundamentals, paired with a publicly observable investment mandate, affect the prices by a factor of 1, the rational revision of the price following persistent demand shocks. The price response is even larger when passive investment flows are correlated with past performance, as illustrated in Section 5.

In the presence of passive investors with stochastic wealth flows the stock market is incomplete, as there is one risky asset and two sources of uncertainty, earnings and flows, that capture the economic fundamentals and the idiosyncratic noise of passive investors, respectively.²³ And hence, in financial markets with passive investors, the stock price cannot replicate the real earnings, motivating the hedging term of the demand of active investors. Passive investments unrelated to earnings news affect asset prices and determine stochastic changes in the opportunity set of active investors. Rather than being uniquely determined by the absence of arbitrage, the risk-neutral measure is uniquely pinned by the preferences of active investors, which require the correction for risk p_γ .

²³New investments into static asset allocation fund raise the utility of those already participating to the stock market, generalizing the result of [Bond and Garcia \(2022\)](#) that higher index investing raises the utility of active investors.

Quantities held by investors characterize the equilibrium in conjunction with the price. The active investors wish to hold a risky asset position that increases in the expected returns per unit of variance and decreases in the risk-free rate and in the wealth managed by passive investors, regarded as a state variable which correlates with the market performance. The passive investors consistently fulfill their mandate. Interestingly, comparative statics with respect to portfolio holdings offer a compelling interpretation to the effects of passive demand on asset prices; let $r \rightarrow \infty$ to lure active investors entirely into the bond market and mute their demand for stocks. When the riskless outside option is infinitely profitable, $X = 0$, and by market clearing $\theta V = P$ must hold. This extreme scenario thus captures price dynamics when the passive investors own the entire stock market, in which case stock prices are simply a unit of account for the wealth they attract, and shares similarities with the cash-in-the-market pricing discussed by [Allen and Gale \(2005\)](#) in relation to active financial intermediaries. The corner case confirms the uniqueness of the equilibrium from a different angle, showing that in any linear pricing function where fundamentals and demand forces have separable effects, the price impact of passively invested wealth is determined by the asset allocation mandate.

Thus, the price impact of trades is tied to the dynamic behavior of the pool of buyers. Following unexpected demand shocks unrelated to prospective fundamentals, the price changes to reflect the entire stream of future demand. The price impact exerted by active investors is instead the force that ensures the equivalence between the left and the right hand sides of Equation (7) to hold at all times. In the making of informed decisions, active investors regard passively invested money itself as information.

Much of the action takes place through the market clearing condition. It is, thus, insightful to inspect the requirement that the number of shares held by active and passive add up to the number of shares outstanding, $X + Q = 1$. This condition, paired with the demand function of passive investors, directly implies that the equilibrium holdings feature the perfect separation given by

$$X_t(P, D, V) = \frac{\text{PDV}_t(D)}{P}, \quad Q_t(P, V) = \frac{\theta V}{P}. \quad (8)$$

These equations show that the stock investment in the portfolio of active investors $X_t(P, D, V)$ are comparatively high when the present discounted value of fundamentals $\text{PDV}_t(D)$ is large, *ceteris paribus*. By the same token, passive investors' shares $Q_t(P, V)$ are comparatively high during market downturns. The equilibrium price presented in closed form ensures consistency between the market clearing condition and the first or-

der condition of the optimization program of the active investors. Economically, active investors have the ability to protect themselves from downside risk and thus reduce their stock positions when higher volatility is not met by offsetting larger expected excess returns. By contrast, for passive investors it does not matter a great deal whether the market moves upwards or downwards, as long as their deviation from their target asset allocation is minimized: hence, passive managers remain invested even in downturns. Proposition 2 shows analytically these properties of portfolio holdings, confirming that active investors reduce their market exposure in periods of high volatility.²⁴ In conclusion, investment positions are of importance for the information content of market prices relative to economic fundamentals. Formally, price informativeness is defined as the forecasting power of prices for future cash earnings (Bai, Philippon, and Savov, 2016). The next result shows its connections with the ownership structure of the market.

Corollary 1. *Price informativeness rises with the incentive of active investors to invest in stocks, as captured by their expected return to risk ratio, and decreases with the position of passive investors.*

Proof. Under the data generating process of Equation (1), the present discounted value of future dividends is the best linear unbiased estimator of cumulative future dividends. Equation (8) shows that the distance between the prevailing market price P_t and the rational forecast of the future value of dividends $PDV_t(D)$ is minimized when active market participants hold the total number of outstanding shares, $X_t = 1$, the corner case considered in Proposition 1. In general, market prices are more informative when active investors have stronger incentives to allocate resources to stocks. From Proposition 2, the position of active investors X_t rises with the Sharpe ratio and decreases in the position of passive investors. Q.E.D.

Intuitively, the information content of asset prices P_t for s periods ahead payoffs D_{t+s} is represented by $PDV_t(D) = p_g + p_D D_t + p_m m$, a linear projection of the current payoff D_t , since the equilibrium has the strong Markov property. Given that $P_t = PDV_t(D) + \theta V_t$, passive investments θV_t systematically bias the information content of asset prices. In order for active investors to be happy to reduce their positions and meet the demand of

²⁴The model generates novel results while maintaining consistency with the patterns recognized in the previous literature. Veronesi (1999) shows that the tendency of stock prices to overreact to bad news when fundamentals are good and underreact to good news when fundamentals are bad is consistent with the learning dynamics of Bayesian agents. The present paper also generates this effect, even if the economic mechanism is entirely different. Equation (8) shows that in good times active investors hold a larger proportion of the outstanding shares and the price thus features higher responsiveness to the fundamentals. All else equal, when the present discounted value of dividends is low the passive investments exert stronger effects on market prices and attenuate their responsiveness to the economic fundamentals.

passive investors, the risk/return trade-off must decrease, uncovering a new relationship between the business conditions and the information content of asset prices.

A contribution of the paper is thus to show that, all else equal, when the risk/return ratio is higher the scope for passive investors' demand to affect asset prices is more limited, since the optimizing agents take comparatively more aggressive positions. By contrast, when the volatility of returns is higher and active investors reduce their exposure, the effect of passive investors' demand on asset prices is larger. In sum, the importance of passive demand for asset prices is time-varying and so is price informativeness. While it appears necessary to consider the empirical content of this result in isolation from trends in technological progress and costs of information gathering, this channel may help to explain why price informativeness drops during the dot-com bubble and the global financial crisis (see [Bai, Philippon, and Savov, 2016](#), Figure 3). This finding shares the interest in relating price informativeness with observable market data of [Dávila and Parlatore \(2023\)](#), who develop a rational expectation equilibrium model with heterogeneous beliefs to study the connection between idiosyncratic volatility and price informativeness. In comparison to their findings, the above result speaks to the aggregate informativeness of the stock market in the time series, and it does so by proposing a novel mechanism based on investor heterogeneity and persistent price deviations from economic fundamentals.

The intertemporal asset pricing structure employed gives rise to rich dynamics of the stock price level P_t , which can be found by applying Itô's Lemma to Equation (7) to obtain

$$dP = p_D dD + \theta dV.$$

Stock market dynamics are driven by the evolution of fundamentals and by the evolution of passive investments. Discount rates are instead fixed by the CARA utility specification and the associated tractable economic environment. It is worth emphasizing that prices may in fact change without the arrival of incremental information about real economic activity. Price revisions are accompanied by changes in the ownership structure which affect the dynamic behavior of prices. This can be seen by replacing in the above expression the law of motion of passively invested wealth dV of Equation (4).

$$dP = p_D dD + \theta [rV(1 - \theta)dt + Q(dP + Ddt) + \pi dF]. \quad (9)$$

Equation (9) highlights one of the central mechanisms of the paper. The capital gain or

losses dP directly affect the wealth of both active and passive investors. But the ownership structure of the market is far from being irrelevant. For ease of illustration, consider the case of capital gains with positive dP , and recall that a proportion $X(P, D, V)$ of shares of the risky asset is held by active investors, and the remaining $Q(P, V)$ by passive investors. When faced with capital gains, active investors become wealthier and increase their consumption level $c(W, V)$. As a direct consequence of their tractable CARA utility function, the investment choice of active investors is not related to their wealth W . Active investors identify and exploit profitable trading opportunities by freely borrowing at the risk-free rate r , regardless of their wealth. In fact, their demand for stocks is always tied to the expected compensation per unit of risk. Active investors would only revise their portfolio conditionally on the arrival of unpriced good news about the economic prospects of the risky asset. By contrast, passive investors automatically reinvest a proportion equal to their asset allocation mandate θ of the appreciation of their portfolio into the risky asset. This reaction function moves prices even more, and leads to amplification dynamics equal in magnitude to $A = \frac{1}{1-\theta Q}$, as shown by a simple manipulation of the previous expression.

$$dP = A \left(p_D dD + \theta [rV(1-\theta)dt + QDdt + \pi dF] \right). \quad (10)$$

This result sharply contrasts with common wisdom. As discussed, the asset allocation funds are contrarians, and must trade in the opposite direction of price changes. Thus, how can passive funds have amplifying effects on market prices? A numerical example clarifies matters.

Example 2. *Suppose that the risky asset trades at $P = \$100$ with ownership equally distributed between active and passive investors, thus $X = Q = 0.5$. Taken together, passive investors wish to invest a share $\theta = 0.4$ of their wealth in the stock market and the remainder in the bond market. As firms announce good earnings, the present discounted value of future dividends increases by $\$20$. In equilibrium, active investors want to hold more shares, and following their heightened demand the price rises to the value of $P + dP = \$120$. Active investors realize capital gains $dW = XdP = \$10$ and increase their consumption level. At the new price, the wealth of passive investors rises by $dV = QdP = \$10$. Of this capital gain, a share $\theta dV = \$4$ is reinvested into the stock market, leading to a price of $\$124$. The higher price has a second-round effect on the wealth of both groups of investors and yields new capital gains, part of which are*

reinvested, and so forth. The amplifying dynamics A are equal to $\frac{1}{1-\theta Q} = \frac{1}{1-0.4 \cdot 0.5} = 1.25$. The new equilibrium price of the risky asset is \$125. Following earnings news which have induced a revision of the fundamentals of \$20, the price has changed by \$25. The ownership of the stock market has changed to $X = 0.56$ and $Q = 0.44$. With respect to the previous Example 1, news about fundamentals are the same, and the only variation is in the ownership structure of the stock market. If the active investors held the entire market, following the same news the risky asset would have closed at the price of \$120.

This example shows that after the upward price pressure the passive fund sold 0.06 shares, which suggest that passive investors moved against the price change. But the true counterfactual in Example 1 was not observed. The data gathered on prices and trades fail to capture the price pressure coming from the shares that passive investors did *not* sell. One key difference between active and passive investors is that active investors realize their capital gains, since their perfectly elastic demand does not depend on own wealth (see, e.g., Veronesi, 1999). Passive investors must instead continuously reallocate their wealth. After stock price surges, the passive fund reduces the *number of shares* held, Q_t . However, as the stock price rose, the higher *wealth* of the fund V_t strengthens the price pressure it exerts. Ultimately, it is the wealth invested which leads to an impact on securities prices. Formally, the adequate measure for price pressure is invested wealth rather than the shares investors trade. This argument raises the bar for empirical studies who focus on investor behavior.

Earlier contributions have focused on the fact that demand unrelated to fundamentals slopes down in the price of stocks (Shleifer, 1986; Kojien and Yogo, 2019). Less attention has been devoted to the elasticity of demand to own wealth. The model suggests that demand curves in fact also *slope upwards in passive investors' wealth*. Formally, Equation (10) demonstrates that the price pressure exerted by investors who allocate wealth in static proportion across asset classes results into wealth and price amplification dynamics.

The reader will recognize the resemblance of wealth amplification effects to prior contributions in the literature. In Kyle and Xiong (2001), convergence traders who are marginal for the valuations of assets in interrelated markets determine their risk premia as a function of their wealth. Wealth effects are also present in Basak and Pavlova (2013), where however there is no flow risk, the market is complete, the proportion of active asset managers is static, and their wealth does not itself feed back into asset price dynamics. The effect modelled here is new to the literature since it applies to a distinct category of market participants, i.e., the investors with static allocation across asset classes. The essence of static investing behavior regards its predictable response to price changes.

The notion of predictability thus naturally relates to the ownership structure of the market. The passively held share of the market *amplifies* price dynamics. Intuitively, the passively held share of the market Q_t is the loading of future price pressure on the stock market. When the passive equity share Q_t is large, price changes lead to substantially higher levels of passively invested wealth which induce upward price pressure. From Equation (8), the proportion of shares held by passive investors Q_t is countercyclical. Moreover, Q_t contains predictive information associated with the prospective reaction of passive investments to market movements. Predictability of the investment decisions of passive funds with asset allocation mandates affects both the first and the second moment of asset price dynamics, as formalized by the next result.

Lemma 1. *Recall that $A_t = 1/(1-\theta Q_t)$ denotes the wealth amplification effect associated with the passive investing style. Time-varying price dynamics can be expressed as follows.*

$$dP_t = \mu_t dt + A_t \left(\frac{\omega}{r} dB_t + \theta \pi dF_t \right),$$

where

$$\mu_t = A_t \left(\frac{m}{r} + \theta [rV_t(1-\theta) + Q_t D_t] \right). \quad (11)$$

Therefore,

$$\sigma_t = A_t \left(\sqrt{\frac{\omega^2}{r^2} + (\theta \pi)^2} \right). \quad (12)$$

Proof. Follows by replacing the dynamics of dividends $dD_t = adE_t$ and passive investments dV into the price dynamics in Equation (10). Q.E.D.

In the more comprehensive equilibrium where passive investments impact market prices, stock price dynamics result from the mixture of two distinct processes, representing economic fundamentals and passive demand for stocks. In more detail, Equation (11) shows that the risky asset's price drift is composed of the discounted earnings drift $\frac{m}{r}$ and the predictable evolution of price pressure, which results from the proceeds of the wealth invested in the bond market $rV_t(1-\theta)$ as well as from the dividends distributed to passive investors $Q_t D_t$. Simply put, strong earnings deliver handsome dividends, a share of which is passively reinvested and contributes to generate upward price pressure. Even more strikingly, when predictable dividends are distributed, their reinvestment by passive

investors generates price pressure. The logic is simply that the foreseeable effects of future dividend distributions and associated investments are already priced – the problem of active investors and the resulting equilibrium price explicitly account for the price dynamics in Lemma 1. Thus, the occurrence of trades might still significantly move prices, even if known in advance.²⁵

The volatility of price changes in Equation (12) responds both to the uncertainty over the evolution of economic fundamentals $\frac{\omega}{r}$, as well as to the risk of wealth flows, a proportion $\theta\pi$ of which may affect asset prices introducing non-fundamental volatility, in line with the empirical findings of Ben-David, Franzoni, and Moussawi (2018), who show that ETF ownership increases volatility and introduces undiversifiable risk. The importance of either of the two forces interacts with the composition of demand, as illustrated by the amplification of both the drift and diffusion by the factor A_t , which quantifies the “feedback loop” between the asset price movements and the investment decisions of the passive investors. When the market is mostly held by passive investors, price movements and volatility thereof are amplified by procyclical price pressure. As the proportion of the risky asset held by passive investors becomes small, instead, the solution approaches the equilibrium in Proposition 1, from which might in general differ as flow risk commands a compensation even in the absence of immanent price pressure.²⁶ The following remarks illustrate notable properties associated with the equilibrium.

Remark 1. *The amplification effect A_t of the passive investing style on the dynamics of the risky asset dP_t varies in the time series. Specifically, these effects are stronger when passive investors own a larger proportion of the stock market Q_t .*

Remark 1 highlights that the magnitude of the reaction of the stock price to innovations, whether regarding news about economic fundamentals or wealth flows to passive investors, depends on the ownership structure of the market. When passive investors hold large shares of the market, the proceeds of upward price revisions are reinvested, resulting in mounting demand pressure that amplifies the price increase. The effect is entirely symmetric. Moreover, when prices deviate from fundamentals the equity valuation

²⁵Lemma 1 can thus parsimoniously explain the findings of Hartzmark and Solomon (2022), who document that days in the top quintile of dividend payments are associated with higher market returns. The amount of dividends is determined ahead of the dividend pay date, and hence the effect documented cannot be ascribed to information. The impact of dividend price pressure has increased since 1990, as passive mutual funds and ETFs have become a larger component of equity holdings. On a related note, Berkman and Koch (2017) document abnormal returns and trading volume around the dividend pay dates on the stocks of firms with dividend reinvestment plans.

²⁶The benchmark equilibrium in the tradition of Merton (1973) illustrated in Section 3.7 achieves as a particular case when the equity allocation θ is set to 0, but does not otherwise result when the wealth of passive investors V_t equals 0, since wealth inflows might still influence the latter. Two more special cases of interest are asset allocation mutual funds, which result when $\theta \neq 1$, and ETFs, that achieve when $\theta = 1$. Equity shares which are negative or larger than one in absolute terms are other conceivable examples, perhaps less frequently observed.

ratio $\frac{P}{E}$ rises in the price pressure exerted by passive investors and features meaningful variation in the time series.

Mutual funds and ETFs are professional asset managers who invest a constant proportion of their wealth into equities. Figure 2 shows one measure of their importance, their ownership share of the Standard & Poor's 500, along with the index real price and earnings recorded over 150 years. The figure shows that the volatility of prices exceeds the volatility of fundamentals, but is still connected to it. As in Remark 1, the stock market ownership share of MFs and ETFs is positively correlated with the responsiveness of prices to earnings in the data. The structural breaks in the price/earnings multiple around 1954 and 1994 first documented by Lettau and Van Nieuwerburgh (2008) are a clear reflection of this amplified dependence of price on earnings, and coincide with persistent changes in the ownership structure.²⁷

Remark 2. *Passive investing style intensifies stock price volatility σ_t .*

Equation (12) illustrates that a higher ownership share passive investors Q_t strengthens the amplification dynamics A_t and thus induces higher price volatility. Volatility reflects two sources of risk, dB for earnings and dF for capital flows, and is both stochastic and predictable, since the ownership structure of the market belongs to the information set of market participants, $Q_t \in \mathcal{F}_t$. After active investors observe a large drop in the stock price, a publicly available signal, their optimal forecast of the distribution of price changes features higher volatility and fatter tails, inducing more conservative portfolio choices. As a result, the model generates volatility clusters. The price dynamics are best considered in combination with the portfolio holdings in Equation (8), according to which the proportion of the market held by active investors is higher when fundamentals are strong, in which case the conditional price volatility becomes lower going forward.

Following standard conventions, μ_t denotes the drift of stock price changes and σ_t denotes their volatility. The drift and volatility of *returns* are obtained by dividing these quantities by the price level, P_t . Thus, in line with previous literature, when the market price is high, both the expected returns and the volatility are low, consistently with the well-documented tendency of stocks with high valuations to have low expected returns.²⁸ Moreover, demand pressure generates asymmetric features in the behavior of volatility,

²⁷Previous explanations have focused on improved capital markets participation (Vissing-Jørgensen, 2002) and the prospects of higher productivity growth (Jermann and Quadrini, 2007). These earlier contributions are consistent with a Gordon model where stock prices are equal to the present discounted value of dividends.

²⁸The exposure of stocks to demand pressure is associated with lower expected returns in related contributions, such as Kojien and Yogo (2019) and Pavlova and Sikorskaya (2023).

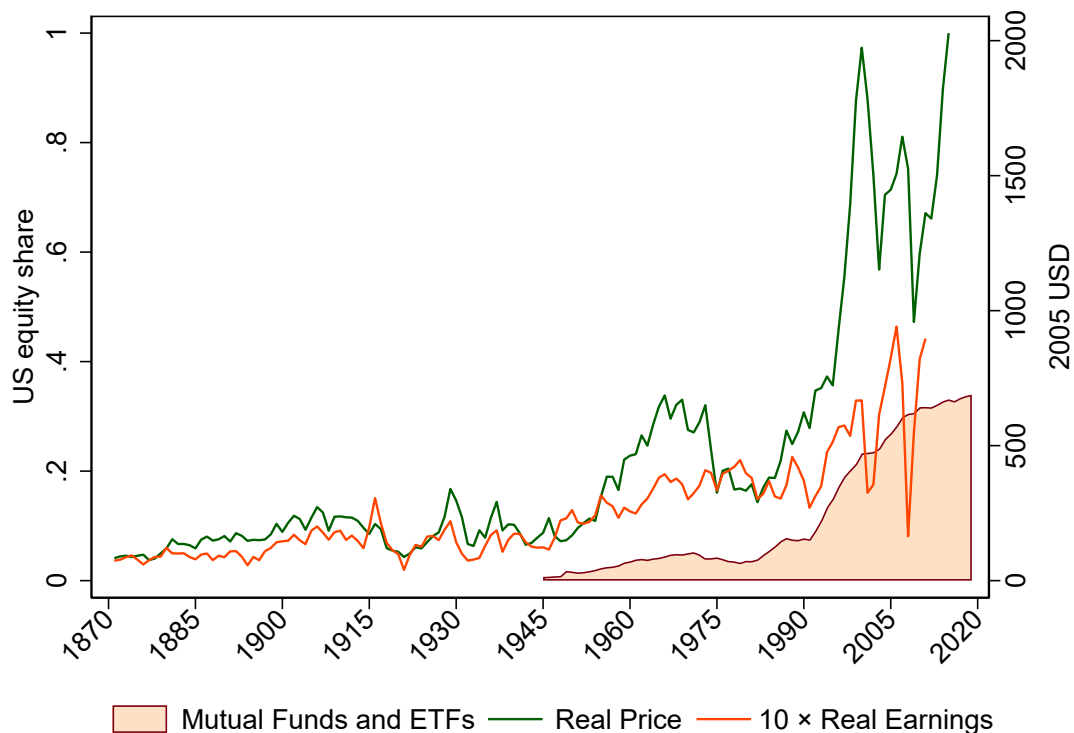


FIGURE 2: **Market Ownership, Price, and Earnings.** The figure shows the U.S. equity ownership share of domestic Mutual Funds and ETFs (left y-axis), as well as the price of the Standard and Poor’s 500 and the earnings accruing to the index (right y-axis), both expressed in real terms. The data are obtained from the Fed Flow of Funds and Robert Shiller’s website.

which spikes when the market tanks.²⁹ Static allocation rules may thus help to explain the asymmetric behavior of volatility. While high market prices have a calming effect on the volatility of returns, this effect is stronger when prices are upheld by fundamentals as opposed to high prices sustained by demand forces.

In the presence of passive investors, stock prices are more sensitive to news when the Sharpe ratio of the equity asset class is low. This occurs because as active investors fly to the safety offered by the risk-free asset, passive investors own a comparatively larger proportion of the market, and thus the procyclical price pressure exerted by the automatic reinvestment of their capital gains and losses becomes more important for asset prices. As a result, the amplification of news resulting from *passive* investors is stronger during financial crises. The importance of *active* intermediaries in the transmission of financial

²⁹This property is sometimes referred to as the leverage effect, because when prices are low firm leverage increases along with uncertainty (Black, 1976). But recently, Hasanhodzic and Lo (2019) have documented that the inverse relation between stock price and return volatility is not specific to firms with leverage.

crises, through risk premia effects and via the financial contagion documented by [Billio, Getmansky, Lo, and Pelizzon \(2012\)](#), is extensively studied in the literature. However, this paper is the first to point out that the stronger amplification of financial fluctuations during downturns also originate from *passive* investors. Intuitively, passive investors remain exposed to the equity asset class even during crises, when their procyclical price pressure becomes more important for the valuation of securities. This result requires a model where the passive investments are priced on the stock market, and the dynamics of the wealth of passive investors is endogenous to asset prices.

Moreover, returns are also more sensitive to flow risk when asset prices are sustained by high demand, as for instance during in the dot-com bubble. As documented by [Adam, Marcet, and Nicolini \(2016\)](#) among others, equity valuation ratios are persistent and have time-varying predictive power for excess returns. Moreover, the volatility of returns depends positively on its past realizations and negatively on stock market prices, consistently with the model's prediction in the presence of passive investors.

3.9 Discussion

The presence of investors that follow static asset allocation strategies is quantitatively large and historically on the rise, and generates wealth amplification effects on asset prices even in a tractable economic environment where the marginal investors are endowed with CARA preferences. These effects arise from investment mandates which specify precise asset allocation rules, and vary endogenously with the composition of the investors. Similar to [Dumas, Kurshev, and Uppal \(2009\)](#), this paper considers the presence of passive investors as a given feature of the financial markets, and examines the optimal investment policy of an intertemporally optimizing investor in equilibrium. The results contributed generate important considerations for the literature.

Markets are efficient when share prices fully reflect the information publicly available, in the definition of [Fama \(1970\)](#). The consensus view in financial economics holds that passive money is not attentive to information, so that if markets were efficient active investors should intervene when prices become unmoored from the present discounted value of fundamentals. However, passively invested money itself represents information which sophisticated market participants must account for when managing their portfolios. The paper highlights that tests for market efficiency should take into account that publicly available information must encompass the characteristics of investors. In practice, ownership data are often available to sophisticated market participants. The paper thus

presents a view of informationally efficient markets. Financial markets may however be inefficient from the allocative viewpoint, particularly taking into account the real effects of stock prices (Bond, Edmans, and Goldstein, 2012).

Equity valuations are closer to the present discounted value of earnings when earnings are stronger, *ceteris paribus*. This feature of the derived equilibrium shares similarities with rational expectations markets with asymmetric information (Grossman and Stiglitz, 1980), where stock prices become less informative about economic fundamentals as the number of active funds and their information gathering decreases. While this paper is similar in spirit, the mechanism of interest springs from the heterogeneous investing styles of market participants, and differs from asymmetric information considerations. This focus on investing style is empirically relevant, as it calls for easily observable data on portfolio holdings and investment mandates, both of which have been extensively documented to have significant effects on securities prices in the empirical research.

4 The Cross Section

The model presented thus far with two groups of agents, active and passive investors, is a characterization of the aggregate stock market in the time series. However, the minimum set of investors necessary to meaningfully characterize the cross section of stocks is composed of *three agents*, which are illustrated in Figure 3. Active investors such as sophisticated households and HFs are the least constrained agents, who select the optimal mix of stocks in their portfolio and also continuously decide on the allocation of their wealth between that mix of stocks and the risk-free asset. Passive investors on aggregate fix a constant equity share in the *time series*, and yet feature significant differences in the *cross section*: active MFs select stocks optimally, but keep a constant proportion of their wealth allocated to the equity market. Passive mutual funds and index-tracking ETFs, which replicate a benchmark index, are passive even in the cross section, and thus constitute the most constrained agents. These differences in the asset allocation mandates of passive investors are motivated by their institutional characteristics, and have an important role for the pricing of the cross section of stocks.

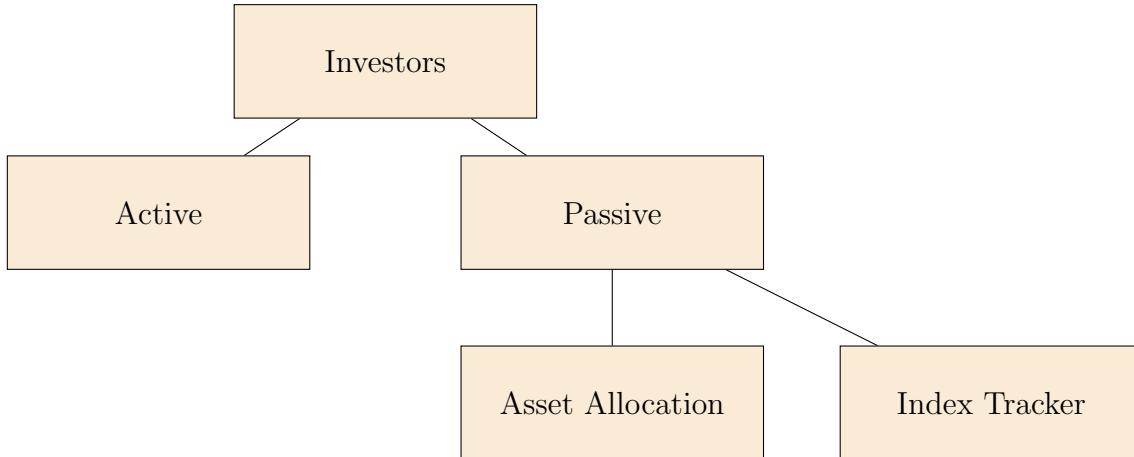


FIGURE 3: **Market Participants.** The figure illustrates the groups of investors relevant for the cross section of stocks. Active investors are unconstrained in the time series and in the cross section of stocks. Passive investors have asset allocation mandates that constrain them to keep constant equity shares over time. In the cross section, asset allocation mutual funds select optimally the stocks in their portfolio, while index-tracking ETFs passively replicate the performance of their benchmark index.

4.1 Assets

There are I firms in the economy. Real earnings E_{it} of firm i at time t follow the dynamics

$$dE_{it} = m_i dt + \omega_i dB_{it}, \quad (13)$$

where m_i is the expected growth of earnings, and ω_i their volatility. B_{it} is a Brownian motion adapted to $\{\mathcal{F}\}$ and the pairwise correlations between earnings news are $dB_{it}dB_{jt} = \rho_{ij}dt$. Companies are publicly listed on the stock exchange, trade at real price P_{it} , and issue dividends D_{it} with constant earnings payout ratio a .

The stock market index summarizes the performance of the economy and can be replicated by a combination of stocks, but is itself not traded.³⁰ Each stock is in fixed supply \bar{S} , normalized to unity without loss of generality.³¹ As a consequence, float-adjusted market capitalization index weights coincide with simple price weights.³² A set of stocks is not included in the index. The dummy variable N_i equals 1 if stock i is included in the index, and 0 otherwise. The index is composed of a subset of firms $N \subseteq I$

³⁰This assumption aims to mirror real-world financial markets and does not affect the results.

³¹Betermier, Calvet, and Jo (2023) consider the decision of corporations to issue stock shares in a production economy.

³²The index can be interpreted as a price-weighted one, such as the Dow Jones Industrial Average. Relaxing the assumption of a constant supply of shares would complicate the derivations without affecting the key results.

stocks and has price

$$P_t^{\text{IDX}} = \sum_{i \in I} N_i P_{it}.$$

The index contains only a subset of stocks. The risky asset class of Section 3, which is the total market value of corporate equities, has price

$$P_t = \sum_{i \in I} P_{it}.$$

The aggregate earnings accruing to the total risky asset portfolio P_t follow

$$dE_t = \sum_{i \in I} m_i dt + \sum_{i \in I} \omega_i dB_{it},$$

which adds up to the earnings of the risky asset class in Equation (1), pinning down the drift and volatility of aggregate earnings,

$$m = \sum_{i \in I} m_i, \quad \omega = \sqrt{\sum_{i \in I} \omega_i^2 + \sum_{i \in I} \sum_{j \in I} \rho_{ij}}.$$

Importantly, this feature ensures the consistency of the analysis of the cross section with the results derived for the aggregate time series. Define by μ_{it} and σ_{it} the price drift and diffusion of stock i and by ρ_{ijt} the correlation of stock prices i and j . Next, consider the portfolio problem of the economic agents.

4.2 Active Investors

The wealth of the active investors follows classical [Merton \(1973\)](#) dynamics

$$dW_t = (rW_t - c_t)dt + \sum_{i \in I} X_{it}[(D_{it} - rP_{it})dt + dP_{it}], \quad (14)$$

with the indirect utility function and transversality condition of Equation (2) unchanged. X_i denotes the fraction of shares of the i -th firm held by active investors.

4.3 Passive Investors

As a group, passive investors wish to invest a proportion θ of wealth in the stock market. However, important differences among allocation investors arise in the cross section. Asset

allocation funds invest a constant share of wealth in equity, but otherwise they act as “*stock pickers*” and optimize their portfolio in the cross section of stocks. Index funds invest a constant share of wealth in equity, and act as “*index trackers*” by weighting each stock in proportion to its contribution to the index. Asset allocation investors are denoted by the superscript A , and index trackers by the superscript IDX . The wealth invested passively on the stock market V discussed in Section 3 is the sum of the wealth of asset allocation investors and index trackers, $V^A + V^{\text{IDX}} = V$.

4.3.1 Asset Allocation Funds

Asset allocation funds such as Asset Allocation MFs and U.S. Equity MFs wish to invest a fixed proportion of their wealth in stocks, but otherwise optimize their portfolio holdings in the cross section of stocks.³³ The asset allocation fund has wealth V^A , invested in respective proportions $1 - \theta$ and θ in the stock market and in the risk-free asset, regardless of the changes in the opportunity set. Asset allocation funds receive a share π^A of the aggregate wealth flows F . Within the equity asset class, the representative asset allocation fund sets its portfolio shares q_{it} optimally. The asset allocation fund has wealth dynamics

$$dV_t^A = rV_t^A(1 - \theta)dt + \theta V_t^A \sum_{i \in I} q_{it} \frac{dP_{it} + (D_{it} - rP_{it})dt}{P_{it}} + \pi^A dF_t. \quad (15)$$

The asset allocation fund takes as given its mandate to allocate a fixed proportion θ of its wealth to the stock market. However, it optimally selects its cross-sectional stock holdings q_{it} as a solution to the following mean-variance portfolio problem.

$$\max_{\{q_{it}^A\}} \mathbb{E}_t[dV_t^A] - 0.5\gamma\mathbb{E}_t[(dV_t^A)^2] \quad \text{s.t.} \quad \sum_{i \in I} q_{it}^A = 1, \quad \theta \text{ given.} \quad (16)$$

Asset allocation investors thus compare risk and returns across stocks, even if their investment decision across asset classes is determined by their mandate θ . In the above, the risk aversion γ that asset allocation investors use to guide their decisions equals that of active investors.³⁴ Effectively, asset allocation differ from active investors only because the proportion of their wealth invested in stocks is fixed, rather than sensitive to changes in the investment set. The fraction of the i -th stock held by asset allocation funds is $Q_{it}^A = q_{it}\theta V_t^A/P_{it}$.

³³U.S. Equity MFs invest about 93% of their wealth in stocks.

³⁴This assumption does not affect the results, and is rather meant to create a level-playing field with the preferences of active investors, ensuring that the financial decisions of asset allocation investors are not driven by their risk preferences.

4.3.2 Index Trackers

Index-tracking ETFs and passive mutual funds have wealth dynamics

$$dV_t^{\text{IDX}} = rV_t^{\text{IDX}}(1 - \theta)dt + \theta V_t^{\text{IDX}} \sum_{i \in I} \lambda_i \frac{dP_{it} + (D_{it} - rP_{it})dt}{P_{it}} + \pi^{\text{IDX}} dF_t. \quad (17)$$

Index trackers invest a constant share θ of their wealth V^{IDX} in the stock asset class and receive a share $\pi^{\text{IDX}} = \pi - \pi^A$ of wealth flows. As an additional constraint, index trackers must invest a fraction of their equity holdings into each stock in proportion to its weight in the index. Index-tracking ETFs and passive investors thus invest into each stock a proportion of their wealth allocated to stocks equal to $\lambda_i = 1/N$, since the benchmark index is price weighted. The fraction of the i -th stock held by the group of index-tracking investors is $Q_{it}^{\text{IDX}} = \lambda_i \theta V_t^{\text{IDX}} / P_{it}$.

4.4 Market Clearing

Each stock is in fixed supply of shares \bar{S}_i , normalized to unity without loss of generality. The fraction of shares of each stock held by active investors is denoted by X_{it} , and the fraction of shares held by passive investors by $Q_{it}^A + Q_{it}^{\text{IDX}} = Q_{it}$. The market clearing condition for the i -th stock is

$$\begin{array}{ccccc} \text{Active Investors} & & \text{Allocation Funds} & & \text{Index Trackers} \\ \downarrow & & \downarrow & & \downarrow \\ X_{it} & + & Q_{it}^A & + & Q_{it}^{\text{IDX}} = \bar{S}_i. \end{array} \quad (18)$$

Proposition 3. *In the cross section, the price of stocks is the sum of the present discounted value of their stream of dividends and of their future passive investments.*

$$P_{it} = \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[\int_t^\infty e^{-r(s-t)} D_{is} ds \right]}_{\text{Fundamentals}} + \underbrace{q_{it} \theta \mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} V_{t+dt}^A \right]}_{\text{Allocation Investing}} + \underbrace{\lambda_i \theta \mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} V_{t+dt}^{\text{IDX}} \right]}_{\text{Index Investing}}. \quad \text{Asset Class Effect}$$

Equity shares and consumption maximizing the Hamilton-Jacobi-Bellman Equation

are

$$X_{it}(P_i, D_i, V) = \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma_{it}^2} - \frac{\sum_{j \neq i} X_{jt}\sigma_{ijt}}{\sigma_{it}^2} - \frac{g'(V)}{r\gamma} Q_{it},$$

$$c_t(W, V) = rW_t + \frac{1}{\gamma}(g(V) + \beta - \log r).$$

Passive investors hold a fixed proportion θ of wealth invested in the equity market and cross-sectional stock holdings equal to

$$Q_{it}^A = q_{it}V_t^A/P_{it},$$

$$Q_{it}^{IDX} = \lambda_i V_t^{IDX}/P_{it}.$$

In the above, index trackers replicate the performance of the index and weight by $\lambda_i = 1/N$ each stock included therein. Asset allocation investors select optimal portfolio shares in each stock q_{it} which satisfy

$$q_{it} = \frac{\mu_{it} - rP_{it} + D_{it}}{\gamma\sigma_{it}^2} - \frac{\sum_{j \neq i} q_{jt}\sigma_{ijt}}{\sigma_{it}^2},$$

$$\sum_{i \in I} q_{it} = 1.$$

Proof. See Appendix B.

The equilibrium in the cross section of stocks is illustrated in Figure 4. Panel A presents a benchmark equilibrium where the only market participants are active investors, who allocate to the optimal mix of stocks a proportion of wealth reflecting the trade-off between risk and return of the equity asset class. Panel B pertains instead to the model with both active and passive investors. Passive investors allocate a constant proportion of their wealth to the equity asset class. Among the group of passive investors, asset allocation investors select stocks optimally.³⁵ Conversely, index-tracking ETFs allocate a constant proportion of their wealth to replicate the performance of the index, which does not include some of the stocks and may thus be inefficient. This investing style of the market participants determines *endogenously* asset prices. While some stocks are overpriced relative to their fundamentals because they are part of an index, certain assets are overpriced relative to their fundamentals because they are part of the equity asset

³⁵Interestingly, the risk-free rate r does not enter analytically the denominator in the stock-picking choice of asset allocation investors q_{it} . The rate r does instead enter the denominator of the investment decision of active investors X_{it} , since these agents can reduce their stock holdings and reallocate their capital to the risk-free asset if its interest rate rises.

class. Both effects are generated by the demand pressure of the passive investors. The demand pressure of passive investors also influences asset price dynamics. In comparison with the benchmark equilibrium, in the equilibrium with active and with passive investors the Sharpe ratio of the equity asset class is lower, and so is the slope of the capital market line. Furthermore, passive investments of index-tracking ETFs lower the Sharpe ratio of the index.

This capital asset pricing framework accounts for investment mandates and departs in several important ways from classical finance theory. In the standard modern portfolio choice setting represented in Panel A of Figure 4, agents decide on the combination of the risk-free asset with the optimal portfolio of risky assets as a function of their risk preferences, and asset price dynamics are exogenous. By contrast, in the model with active and passive investors illustrated in Panel B, asset price dynamics are endogenous to the wealth of passive investors. This occurs because passive investors operate under a mandate, whereby not only they exert price pressure today, but it is also reasonable to expect that they will do so in the future. Therefore, active investors do not have the incentive to take the opposite side of their demand by selling stocks short and investing at the risk-free rate. Take as an example a stochastic change in the investment opportunity set, which reduces the slope of the capital market line and the motive to invest in stocks. Active investors fly to the safety of the risk-free asset, but passive investors remain tied to the equity asset class to by their allocation mandate. Passive investors thus exert a price pressure, in this example by cushioning stock prices.

In general terms, passive investors exert a price pressure on the assets included in their mandate. The price of the i -th stock in Proposition 3 can be compactly expressed in closed form as follows.

$$P_{it} = \text{PDV}_t(D_i) + \theta[q_{it}V_t^A + \lambda_i V_t^{\text{IDX}}], \quad (19)$$

where $\text{PDV}_t(D_i) = p_\gamma + p_D D_{it} + p_m m_i$ is the present discounted value of dividends distributed by stock i and incorporates risk corrections. The resulting individual stock price drift, volatility, and pairwise correlation are

$$\mu_{it} = \frac{\frac{m_i}{r} + \theta[rV_t(1 - \theta) + Q_{it}D_{it}]}{1 - \theta Q_{it}}, \quad \sigma_{it} = \frac{\frac{\omega_i}{r} + \theta\pi}{1 - \theta Q_{it}}, \quad \sigma_{ijt} = \frac{\frac{\rho_{ij}}{r^2} + (\theta\pi)^2}{(1 - \theta Q_{it})(1 - \theta Q_{jt})}.$$

Equation (19) clarifies that the price of each stock equals its discounted future fundamentals plus two components: (i) preferred demand for the equity asset class; and (ii)

the effect of index inclusion. An asset class effect thus arises, over and above the index inclusion effect documented in previous studies. The asset class effect is important even for stocks not included into any benchmark, whose price becomes higher in the presence of static allocation rules.

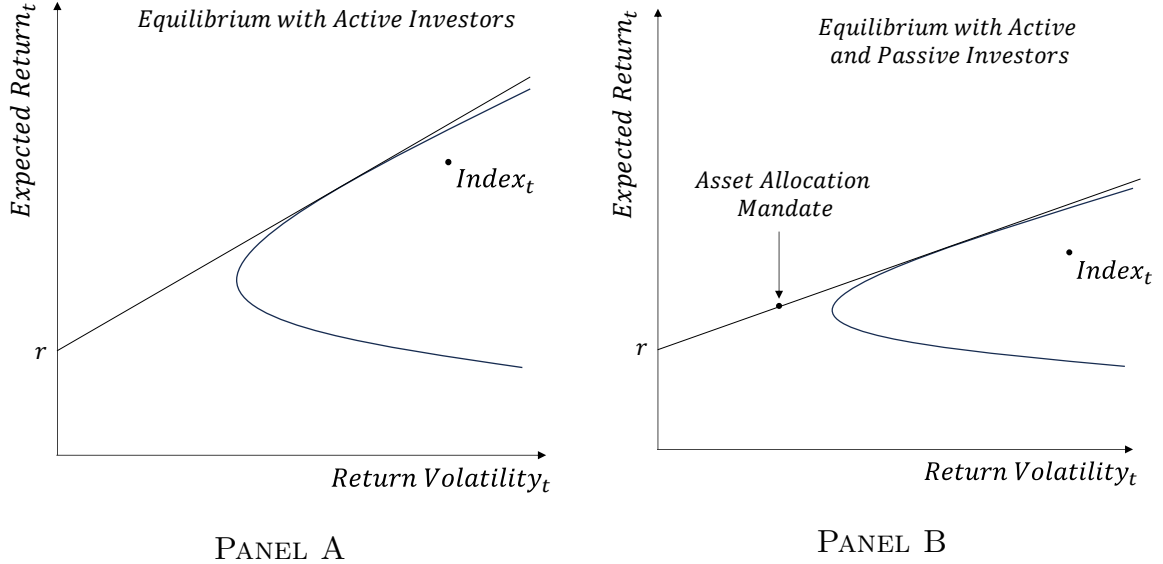


FIGURE 4: **Portfolio Selection of Heterogeneous Investors.** Panel A of the figure illustrates the cross section of stocks in the model with active investors, who allocate to the optimal portfolio of stock a proportion of wealth reflecting the trade-off between risk and return of the equity asset class. Panel B illustrates the model with both active and passive investors. Passive investors allocate a constant proportion of their wealth to the stock market. Among the passive investors, asset allocation mutual funds select optimally the stocks to hold, while index-tracking ETFs replicate the performance of the index. Passive investments increase the prices of stocks, reduce their expected returns, and increase their volatility and pairwise correlations.

The analysis of stock prices and asset allocation mandates in the cross section suggests that the mean-variance efficient frontier is affected by the wealth of asset allocation investors and index-tracking ETFs. The market is incomplete, since changes in the investment opportunity set result from the realizations of the $N + 1$ stochastic processes for earnings and wealth flows. Interestingly, demand forces influence correlations in excess of the fundamentals between pairs of stocks included in the index, a classical result, as well as between non-index stocks and both index and non-index stocks – as shown by the expression for σ_{ijt} .

Expected returns and volatility change over time with the demand of passive investors, which affects the investment opportunity set. A simple way to think about it is that passive investments increase the stock price, reduce the expected returns, and increase

price volatility. The implication is that when the asset allocation mandate θ rises, the investment opportunity set available to each market participant *changes*.

The index inclusion effect depends on the wealth of the index trackers, and has been documented extensively. The *asset class* effect proposed by this paper is an important generalization of the index inclusion effect. Differently from index inclusion considerations, which pertain to the relative pricing of stocks included and not included into a benchmark index, the asset class effect matters for the aggregate pricing of corporate equities relative to the Treasury market. Proposition 3 shows that the wealth invested by investors with fixed asset allocation influences the dynamics of the entire set of securities included in the stock market asset class. Intuitively, this effect relates to the wealth which floods the stock market and is unrelated from risk and returns considerations.

The asset class effect was illustrated in relation to the equity market, but fixed asset allocations are associated with many asset classes or subsets thereof and have obvious implications for market segmentation. For example, the asset class effect could be used to explain the presence of local risk factors documented empirically by [Chaieb, Langlois, and Scaillet \(2021\)](#). As an extension, Section 5.2 sketches a discussion of the consequences of asset allocation mandates for the Treasury market, borrowing a simplified price pressure structure from [Greenwood and Vayanos \(2014\)](#).

5 Extensions and Generalizations

5.1 Flows and Past Performance

To this point, the model has assumed for simplicity that the wealth flow F can be described as a Brownian motion. However, a large literature documents that capital flows are associated with past fund performance.³⁶ To extend the model in this direction, let Z be the process for fund flow which follows the more general dynamics

$$dZ_t = \alpha_t dt + \pi dF_t, \tag{20}$$

³⁶For example, [Lou \(2012\)](#) forecasts the flows in and out of mutual funds using the past performance of the fund and shows that aggregate flows generate predictable excess returns; see also [Franzoni and Schmalz \(2017\)](#). More recently, [Parker, Schoar, and Sun \(2023\)](#) show that Target Date Funds reduce trend-chasing in aggregate equity fund flows.

where the expected flow α_t depends on the entire history of past earnings surprises,

$$\alpha_t = \int_{-\infty}^t \psi e^{-\kappa(t-s)} dB_s. \quad (21)$$

α being a weighted average of the past performance of the real economy with weights ψ that decay at rate κ as shocks occur further in the past (see, e.g., [Maxted, 2023](#)). Appendix C.2 extends the model in this direction, and finds that the main findings are robust and even reinforced.

[Moskowitz, Ooi, and Pedersen \(2012\)](#) document a strong pattern of persistence in returns. While robust across markets and asset classes, at first sight this phenomenon is not easy to reconcile with rational agents operating in markets free of arbitrage opportunities. This evidence is thus often interpreted through the lens of behavioral models, with few exceptions including [Vayanos and Woolley \(2013\)](#). However, given the institutionalization of capital markets, the proportion of investors who are prone to make mistakes should be gradually shrinking over time. In a comprehensive setup that accounts for the presence of passive investors and their effect on securities prices, this result appears more natural. Successive runs of positive and negative earnings are consistent with the effect of current capital gains or losses in generating predictable price pressure through the wealth passively invested in the future. The passive fund attracts higher wealth inflows after the market appreciates. Therefore, market movements affect future price pressure. There are two consequences of this effect. First, market movements generate stronger amplification effects, since the present value of future price pressure reacts more than the current level of demand. The second consequence is on expected returns: if the market price rises, so does the expected change in passively invested wealth dV along with the prospects of price pressure in the future. The effect is completely symmetric, as when the market tanks, passive wealth is set to decrease, reducing expected returns in the future. The correlation between flows and past performance thus generates, or perhaps strengthens, time-series momentum dynamics.

5.2 The Treasury Market

Thus far, the analysis abstracts from the effects of market forces on the riskless asset. However, the bond market is itself subject to price pressure from investors with downward sloping demand ([D'Amico and King, 2013](#); [Vayanos and Vila, 2021](#)). [Greenwood](#)

and Vayanos (2014) focus on the effects of clienteles between the Treasury market and other markets on the term structure. Previous contributions in the literature who focused on the excess comovement between stocks and bonds, such as Shiller and Beltratti (1992), Connolly, Stivers, and Sun (2005), Baele, Bekaert, and Inghelbrecht (2010), David and Veronesi (2013), and Duffee (2023), abstract from the price pressure that results from asset allocation strategies. Asset allocation strategies require to invest a fixed proportion of wealth in bonds, and thus exert price pressure on the Treasury prices. The wealth of passive investors reflects the developments of the stock price, and thus generates comovement between the two markets. Appendix C.2 extends the model in this direction.

There are two possible ways to endogenize the Treasury market. The first approach is to impose the market clearing condition on the risk-free asset. The second one is to use a model with price pressure on the Treasury market, which has the advantage that the demand of passive investors for bonds is not sensitive to news, and constitute a demand risk factor. Greenwood and Vayanos (2014) suggest that a shock to the demand factor should move the yields of all bonds in the opposite direction as the shock and the instantaneous expected returns of all bonds in the opposite direction as the shock. After an exogenous increase to the wealth passively invested, one should thus observe lower yields and bond expected returns. Figure A.3 provides evidence suggestive of the spillover of wealth effects between the U.S. equity and Treasury bond market using dividend pay dates as a clean instrument for wealth shocks unrelated to information. Days with large dividend payment amounts feature large returns on the stock market (Panel A), with low term premia (Panel B), and low expected returns on 10-year Treasury bonds (Panel C).

6 Empirical Estimation

6.1 The Model in the Data

As discussed in Section 3, the model has two asset classes: one is safe and its price is exogenous, and the other distributes stochastic dividends and trades at an endogenous equilibrium price. There are two groups of agents – active investors, who adjust their portfolio holdings in response to news announcements; and passive investors, who follow static allocation rules across asset classes. The model generates three main empirically testable hypotheses.

Hypothesis 1: ASSET CLASS EFFECT. When the wealth invested under the equity allocation mandate increases, the value of corporate equities rises, for constant current

and future earnings and discount rates. Proposition 2 forms the basis for this hypothesis.

Hypothesis 2: WEALTH AMPLIFICATION EFFECT. Asset allocation mandates generate conditional price volatility in excess of the volatility of earnings, but still connected to it. This hypothesis follows from Lemma 1.

Hypothesis 3: PRICE INFORMATIVENESS AND THE RISK/RETURN TRADE-OFF. Asset allocation mandates reduce the information content of stock prices for future earnings, the more so the lower is the expected return/risk ratio. This hypothesis stems from Corollary 1.

It is briefly explained here how the elements of the model and the ensuing analysis help to set up to the empirical examination that follows thereafter. An increase in the wealth invested under the equity allocation mandate increases the demand for the risky asset class in the present as well as the forecast of its realization in the future, generating the asset class effect outlined in Hypothesis 1. Stock price movements affect the wealth of investors with allocation mandates, who reinvest their capital in static proportions and amplify the price responses to news, leading to Hypothesis 2. The price of a stock contains information on its earnings expected to accrue in the future, as well as on its expected future price. Hypothesis 3 follows from the fact that an increase in the wealth committed to the risky asset puts upward pressure on its price and biases its signal for prospective fundamentals, even more so when low and volatile expected returns induce active investors to invest less in the risky asset and more in the safe asset.

Section 4 extends the model to the cross section of stocks in the risky asset class.³⁷ The hypotheses outlined can thus be tested *both* in the time series and in the cross section.

6.2 The Time Series

The theory discussed so far underscores that adequate modeling of asset price dynamics should account for the ownership structure of the stock market. As a first descriptive step of the analysis, consider a yearly data sample retrieved from Robert Shiller's website and the Fed Flow of Funds ranging from 1870 to 2021.³⁸ To gain intuition of the data, it is helpful to construct a *passive share* variable, defined as the proportion of the U.S. stock market held by MFs and ETFs.³⁹ Figure 5 illustrates the positive time-series association between the passive share and the price/earnings ratio, with linear correlation of 0.61. A

³⁷This extension considers three groups of agents: active investors, who respond to news by adjusting their portfolio share of risky assets as well as the stocks composing it; asset allocation investors, who follow static asset class constraints but select stocks optimally; and index-trackers, subject to portfolio constraints both at the asset class and at the stock level.

³⁸Flow of Fund equity data are obtained from Table L.224 at <https://www.federalreserve.gov/releases/z1>.

³⁹The passive share variable is set to zero before 1951, when holdings data are available from the Flow of Funds.

first inspection of the data thus indicates that the passive ownership share of the market correlates positively with the equity valuation multiple. In the model, this occurs since passive investments affect prices, but not earnings. This simple correlation analysis goes in the direction outlined in Hypothesis 1.

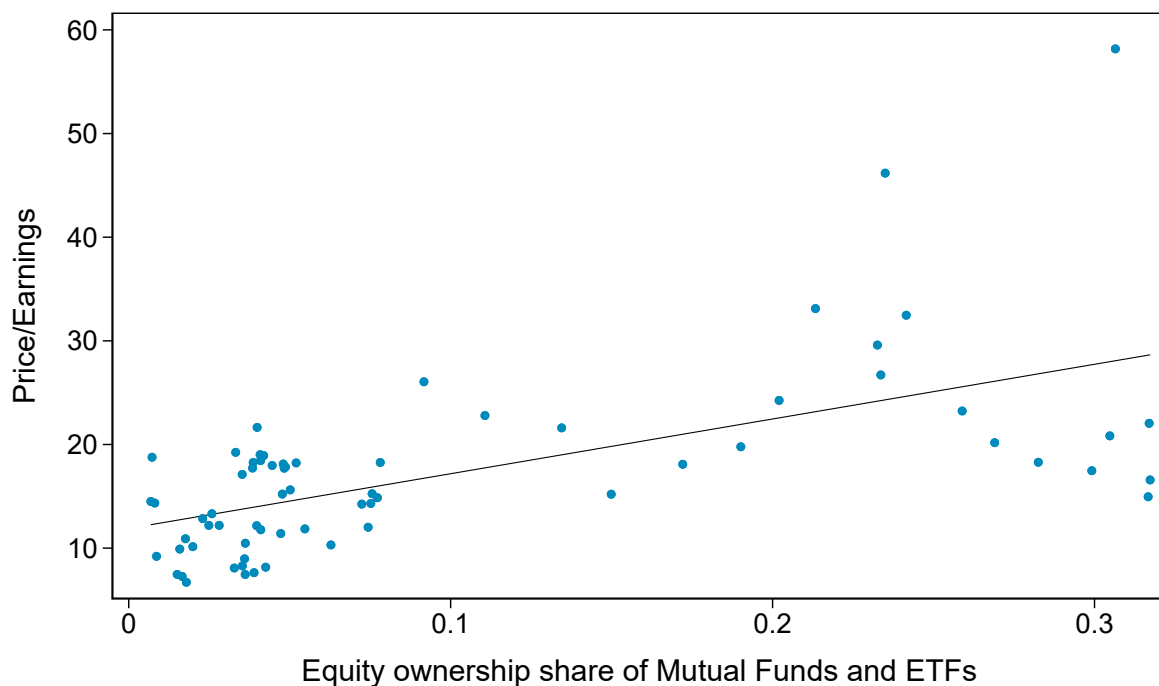


FIGURE 5: **Passive Investing and Equity Valuations.** The figure shows a time series scatter plot of the price/earnings valuation multiple of the S&P500 and the equity ownership held by domestic Mutual Funds and ETFs, using yearly data from the Fed Flow of Funds and Robert Shiller’s website ranging from 1870 to 2020.

In terms of time series implications, a contribution of this paper is to suggest that the same news may have different effects on returns depending on the *ownership structure* of the stock market – see Hypothesis 2. For example, the model suggests that unexpected good earnings should have a stronger impact when they lead to capital gains that are automatically reinvested. These consideration motivate a time series model of volatility with a structural interpretation of equity ownership data.⁴⁰

Consider the GARCH-MIDAS specification for the volatility of stock returns proposed by [Engle, Ghysels, and Sohn \(2013\)](#), that blends a slow-moving component recorded at low frequency and a high-frequency conditionally autoregressive component. The model

⁴⁰To appreciate the novelty, recall that this specification is usually employed to evaluate the effect of macroeconomic variables on market volatility.

reported in Equation (22) relates the returns $r_{d,q}$ realized on day d to a constant mean m , as well as to white noise innovations $e_{d,q}$ that enter the specification through a component model for volatility. The long-run component l_q is a function of the contemporary and lagged proportion of the U.S. stock market held by MFs and ETFs recorded on quarter q , where n is the intercept and f_k is a beta function weighting the K lags included. The short-run component is a GARCH(1,1) model with daily lagged innovations and parameters a and b .

$$\begin{aligned} r_{d,q} &= m + \sqrt{l_q} g_{d,q} e_{d,q}, \\ l_q &= n + c \sum_{k=1}^K f_k(w_1, w_2) \text{Passive Share}_{q-k}, \\ g_{d,q} &= (1 - a - b) + a \frac{(r_{d-1,q} - m)^2}{l_q} + b g_{d-1,q}. \end{aligned} \quad (22)$$

The model in Equation (22) enables the same news to have different effects depending on the ownership structure of the stock market, captured by the proportion of the U.S. stock market held by MFs and ETFs in quarter q and denoted by Passive Share_q – the same variable used in the previous Figures 2 and 5. The model of Section 3 suggests that the response of returns to news should be amplified when passive investors hold a larger proportion of the stock market. In the above specification, the wealth amplification effect is directly tied to c , that is thus expected to be positive and statistically significant.

Holdings data are readily available at the quarterly frequency from the Federal Reserve Flow of Funds statistics, and can be useful to complement workhorse time-series models of the daily volatility of aggregate returns using mixed data sampling techniques. The daily S&P 500 returns data are retrieved from Bloomberg. Table 1 presents the estimates. Panel A pertains to the baseline estimation, and Panel B presents a robustness test where the long-run component is estimated on a rolling basis. The coefficient estimate c has the expected sign and is both statistically significant and economically meaningful. For the full sample, the parameter estimate is 0.024 with a t-statistics of 4.7, suggesting that an increase in passive ownership predicts greater volatility in the financial market for the upcoming quarter. The estimates appear remarkably robust across specifications. For example, in Panel B the estimate of c is again 0.024, with t-statistics of 4.6. Consistently with the proposed theory, where the amplification effect is a concave function of passive ownership, the estimate of c is larger in the earlier 1953-1984 subsample characterized by lower levels of passive ownership. On the other hand, during the 1985-2010 period

the passive ownership variable rises dramatically from 0.06 to 0.31, in correspondence to a long-run component coefficient estimate of 0.014.⁴¹ The results of this analysis of volatility go in the direction outlined in Hypothesis 2.

Aggregate patterns suggest that rising passive ownership is associated with a higher sensitivity of stock prices and returns to fundamentals, but should be interpreted cautiously. Further econometric analyses can be carried out in the cross section of stocks, which has the advantage of delivering a better identification.

6.3 The Cross Section

Wealth amplification effects To further examine the link between stock price returns and ownership structure outlined in Hypothesis 2, this paper relies on cross-sectional regressions of abnormal returns on standardized earnings surprises around corporate announcements. Event studies around earnings announcements are a widely used empirical strategy in financial economics. These studies focus their attention on a narrow window around the event date. Take as an example [Hotchkiss and Strickland \(2003\)](#), who document that the investor composition matters for the response of stock prices to corporate earnings announcements. Consider a baseline panel regression model of the following form:

$$\begin{aligned} \text{Abnormal Return}_{it} &= b_0 + \text{Firm FE} + \text{Time FE} + b_1 \times \text{Earnings Surprise}_{it} \\ &+ b_2 \times \text{Earnings Surprise}_{it} \times \text{Wealth Benchmarked}_{it} + \varepsilon_{it}. \end{aligned} \quad (23)$$

A unit of observation is an announcement earnings of firm i at time t . For each stock, the abnormal return is estimated with respect to the constant mean model, the market model (CAPM), and the [Fama and French \(1992\)](#) model (FF3). Earnings surprises are calculated by taking the increase in earnings per share over four quarters and dividing it by its eight-quarters rolling standard deviation, and wealth benchmarked to each stock is the measure of [Pavlova and Sikorskaya \(2023\)](#). The sample construction follows standard conventions, and is described in detail in Appendix E. The working sample is a comprehensive cross section of more than 5 thousand U.S. firms observed from 1998 to 2018. Table 2 reports summary statistics of firm-level variables. In the data, earnings surprises and benchmarked wealth have average values equal to 0.29 and 0.18, respectively.

⁴¹Future research could assess the forecasting performance of ownership data for volatility, using the data sampling methods discussed in [Ghysels, Plazzi, Valkanov, Rubia, and Dossani \(2019\)](#).

However, earnings surprises have a volatility of 0.25, much higher than the volatility of benchmarked wealth, equal to 0.08.

The event study setup helps to pin down whether the amplification mechanism results from passive wealth. Table 3 presents the results of the estimation. The effects of earnings announcements are amplified by the wealth benchmarked to the stock. The documented effect is economically large and statistically significant, as the baseline earnings response coefficient of 0.276 increases to 0.380 at the median of the distribution of benchmarked wealth, equal to 0.181. The result is robust to alternative statistical models for normal returns. This economically sizeable effect is not easy to explain using standard theories. For example, most theories of overreaction to news about fundamentals are based on the active portfolio choices of extrapolating investors, and remain silent as to why *passive* investors would amplify prices response to news. In the model, this excess sensitivity of prices to earnings news is associated with a wealth amplification effect that originates from the procyclical price pressure exerted by passive investors. This finding is corroborated by Sammon (2022), who uses a different sample and measure of passive ownership and documents that a stock in the 90th percentile of passive ownership responds nearly 3 times as much to earnings news as a stock in the 10th percentile of passive ownership. To assess if the effect is persistent, Panels B and C of Table 3 assess the cumulative abnormal reaction of stock prices over progressively the longer time horizons of 3 and 7 trading days around the corporate earnings announcements. The magnitude of the coefficients is remarkably stable, and while standard errors progressively widen with the event window, the estimates remain statistically and economically significant in all specifications.

Price Informativeness and Passive Investing Hypothesis 3 suggests that asset allocation mandates reduce the information content of stock prices about future earnings, as well as their expected return/risk ratio. The Russell yearly reconstitution offers a clean identification to test whether passive investments affect the information content of stock prices about subsequent accrued earnings. At the end of each May, Russell 3000 stocks are ranked by their market capitalization. The top 1000 stocks are assigned to the Russell 1000, the bottom 2000 stocks to the Russell 2000, and the index composition is fixed for the subsequent year. These indexes are market-cap weighted, and stocks that randomly end up in the Russell 2000 become the largest among small caps and experience a tenfold increase in their index weight, as illustrated in Figure A.4.

The discontinuity in the index weighting around the inclusion cutoff is known to be a clean identification setup. As a result of the yearly index reconstitution, the inclusion of

a stock in the Russell 2000 attracts large passive investments associated with permanent price pressure. The literature consistently identifies positive returns in June without subsequent reversals in the following months (Madhavan, 2003). According to Hypothesis 3, the price response to passive investing should reduce the information content of stock prices about the future realizations of earnings. This can be tested by examining whether the random price changes of stocks included in the Russell 2000 are correlated with higher earnings per share accrued in the year between the inclusion in the index and the following reconstitution on the next year.

The sample construction and methodology employed closely follow Chang, Hong, and Liskovich (2015), who exploit this quasi-natural experiment to identify the price effects of passive investments. The list of index constituents is hand-collected from Bloomberg, the number of shares outstanding is obtained from CRSP, and earnings data are from I/B/E/S. The data range from 1996 up to 2006, after which Russell Inc. adopted a banding policy aimed at reducing the number of index addition and deletions, reducing the accuracy of identification (see also Ben-David, Franzoni, and Moussawi, 2018). This dataset enables to cleanly assess the effect of passive investments, causal in proximity of the reconstitution cutoff, on three key variables of interest – stock returns, ex-post earnings, and the risk/return trade-off.

Figure 6 shows that the price changes caused by passive investments in proximity of the threshold are not predictive of high future earnings. The top panel of the figure presents the returns of stocks in June against their market capitalization ranking at the end of May and confirms the findings of the previous literature that the inclusion in the Russell 2000 is associated with a statistically significant effect on returns. The bottom panel of the figure shows firms' reported earnings during the year following the index reconstitution against their market capitalization rank. The figure thus suggests that the price pressure associated with inclusion in the Russell 2000 is not on account of high future earnings. The null hypothesis that earnings following inclusion are on average higher in the Russell 2000 than in the Russell 1000 is rejected at all standard significance levels. If anything, earnings are higher on the opposite side of the threshold, confirming the positive relationship between firm size and profitability studied among others by Hou and Van Dijk (2019). In sum, price movements caused by passive investments do not predict fundamentals and reduce the information content of stock prices. This experiment supports the first part of the hypothesis.

Moreover, Hypothesis 3 suggests that stocks with higher passive ownership and lower price informativeness should have a lower trade-off between expected return and risk, thus

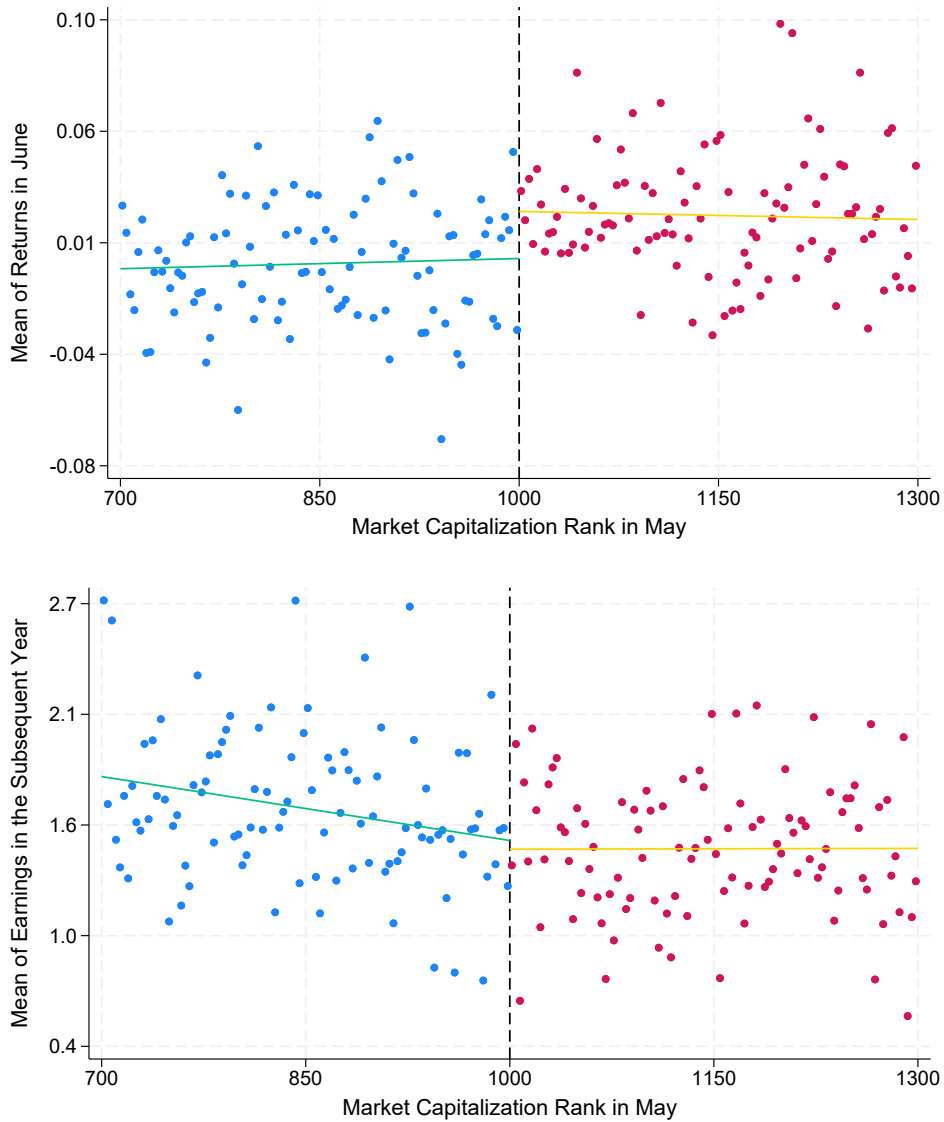


FIGURE 6: Passive Investing and Price Informativeness around the Russell Cutoff. The figure shows the market capitalization rank of the Russell 3000 Index constituents against 100 bins of two variables, both averaged over time: the returns in the month following the reconstitution of the index (panel A), and the sum of the earnings accrued in the year after the reconstitution (panel B). Data from 1996 to 2006 are obtained from Bloomberg, CRSP, and I/B/E/S.

optimally crowding out active investments. The task of measuring ex-ante expectations of risks and returns is not an easy one. On average, however, these expectations can be approximated by the ex-post realizations. The risk/return trade-off of a stock is thus computed as the realized Sharpe ratio, the ratio between the mean of its monthly returns in excess of the risk-free rate over the 12 months between two consecutive index reconstitutions and their volatility. At the end of this procedure, each inclusion and deletion of a stock from the Russell 2000 is linked with the realized Sharpe ratio of the stock in the subsequent year. The average Sharpe ratio realized by stocks deleted from the Russell 2000 in the subsequent year is -0.10, significantly higher than that of stocks included in the Russell 2000, equal to -0.16. A t-test confirms that this difference in means is statistically significant at all standard levels.

The amplification of stock price responsiveness to news and the reduction in price informativeness are two sides of the same coin. By its nature, the accounting system recognizes information with a lag with respect to the stock market. Hence, when the stock price is less informative about future earnings, its responsiveness to corporate earnings reports is higher. Overall, the data are consistent with the predictions of the model.

7 Conclusion

Equity market research has documented systematic patterns in the rise of passive investing and the valuation of securities that are not easy to reconcile with classical asset pricing theories based on the principle of optimal portfolio choices. The existing literature lacks a coherent theory to reconcile the heterogeneous investing patterns resulting from asset allocation mandates with intertemporal models derived from first principles that account for economic fundamentals. This paper has derived a tractable description of securities markets where optimizing agents respond to earnings and to participants whose asset allocation mandate is not related to fundamental news, but to the objective of maintaining constant their exposure across asset classes. The equilibrium was achieved by characterizing the relationship between the composition of investors and the dynamics of asset prices with a particular attention to the economic fundamentals. This undertaking brings several benefits. It highlights that as the passive share of the market rises, equity valuation multiples rise, and stock prices reacts more strongly to news. Moreover, it underscores that the financial markets produce more precise information about the economic fundamentals underlying a security when such security has a favorable expected return per unit of standard deviation. The paper has developed the notion of *asset class*

effect, whereby the value of corporate equities is at least as high as the wealth allocated to the equity asset class through static mandates. This notion is a generalization of the familiar index inclusion effects. In analogy to stocks included into a benchmark index that are overpriced relative to their discounted dividend stream, certain securities are overpriced relative to their discounted cash flows in association with their inclusion in the equity asset class, targeted by the allocation mandate of professional investors.

For ease of presentation, the paper has followed the standard approach in considering the amount of stocks outstanding as exogenous and constant. Nonetheless, corporations consistently issue new shares when market prices are high, and repurchase existing ones in the opposite conditions. Future extension to assess the corporate finance implications of fixed investing rules remain an active area of research. For example, the capital structure of the firm may be relevant for its valuation in the presence of asset class effects. Furthermore, future research could use the unified setup derived in this paper in combination with the effects of preferences of active investors, their leverage, credit, and regulatory constraints which they are subject to, with the purpose of examining the effects of these features on the financial markets in the face of the increasing importance of delegated portfolio management.

Appendix

A Proof of Proposition 2

As is standard in the literature, the proof proceeds by postulating that the price function is as guessed and verifies that the Hamilton-Jacobi-Bellman (HJB) equation, the market clearing condition, and the transversality conditions are satisfied by the candidate formulation.

$$P_t = p_\gamma + p_D D_t + p_m m + \theta V_t.$$

By definition, $p_\gamma = -\frac{1}{r}(\frac{\omega}{r} + \theta\pi)^2$, $p_D = \frac{1}{r}$, $p_m = \frac{1}{r^2}$. Using Itô's Lemma,

$$dP_t = p_D dD_t + \theta[rV_t(1 - \theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t].$$

The state variables follow dynamics

$$\begin{aligned} dD_t &= mdt + \omega dB_t, \\ dV_t &= rV_t(1 - \theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t, \end{aligned}$$

Price drift and diffusion are, respectively,

$$\mu_t = \frac{\frac{m}{r} + \theta[rV_t(1 - \theta) + Q_t D_t]}{1 - \theta Q_t}, \quad \sigma_t = \frac{\frac{\omega}{r} + \theta\pi}{1 - \theta Q_t}. \quad (24)$$

The Hamilton-Jacobi-Bellman (HJB) equation is

$$\begin{aligned} 0 &= \max_{\{c, X\}} U(c) + \frac{\mathbb{E}_t[dJ]}{dt} \\ &= \max_{\{c, X\}} U(c) + J_t + J_W \mathbb{E}_t[dW] + J_V \mathbb{E}_t[dV] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW^2] + \frac{1}{2} \mathbb{E}_t[dV^2] + J_{WV} \mathbb{E}_t[dW dV]. \end{aligned}$$

Moreover,

$$\begin{aligned}
\mathbb{E}_t[dW] &= [rW - c + X(\mu - rP + D)]dt, \\
\mathbb{E}_t[dW^2] &= \mathbb{E}_t[(XdP)^2] = (X\sigma)^2dt, \\
\mathbb{E}_t[dV] &= [rV(1 - \theta) + Q(\mu + D)]dt, \\
\mathbb{E}_t[dV^2] &= (Q^2\sigma^2 + \pi^2)dt, \\
\mathbb{E}_t[dVdW] &= XQ\sigma^2dt.
\end{aligned}$$

By substituting the above expressions in the HJB equation,

$$\begin{aligned}
0 &= \max_{\{c,X\}} U(c) + J_t + J_W \mathbb{E}_t[dW] + J_V \mathbb{E}_t[dV] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW^2] + \frac{1}{2} \mathbb{E}_t[dV^2] + J_{VW} \mathbb{E}_t[dWdV] \\
&= \max_{\{c,X\}} U(c) + J_t + J_W [rW - c + X(\mu - rP + D)] + J_V [rV(1 - \theta) + Q(\mu + D)] \\
&\quad + \frac{1}{2} J_{WW} X^2 \sigma^2 + \frac{1}{2} J_{VV} (Q^2 \sigma^2 + \pi^2) + J_{WV} X Q \sigma^2.
\end{aligned}$$

The first order conditions (FOCs) are

$$\begin{aligned}
U'(c) &= J_W, \\
X &= -\frac{J_W}{J_{WW}\sigma^2}(\mu - rP + D) - \frac{J_{WV}}{J_{WW}\sigma^2}Q\sigma^2.
\end{aligned}$$

Active investors feature CARA utility, suggesting an educated guess for a value function of form

$$J(W, V, t) = -e^{-\delta t - r\gamma W - g(V) - \beta} \quad (25)$$

thus, $J_t = -\rho J$, $J_W = -r\gamma J$, $J_{WW} = (r\gamma)^2 J$, $J_V = -g'(V)J$, $J_{VV} = (g'(V)^2 - g''(V))J$, and $J_{WV} = r\gamma g'(V)J$. Therefore, the FOCs become

$$\begin{aligned}
c(W, V) &= rW + \frac{1}{\gamma}(g(V) + \beta - \log r), \\
X(P, D, V) &= \frac{\mu - rP + D}{r\gamma\sigma^2} - \frac{g'(V)}{r\gamma}Q.
\end{aligned}$$

At this stage, it is standard to replace in the HJB the FOCs paired with the usual market clearing condition $X = 1$. However, the market clearing conditions requires $X + Q = 1$.

By replacing the expression for Q ,

$$X(P, D, V) = \frac{P - \theta V}{P}, \quad Q(P, V) = \frac{\theta V}{P},$$

or, equivalently,

$$Q(P, V) = \frac{\theta V}{p_\gamma + p_D D + p_m m + \theta V}, \quad X(P, D, V) = \frac{p_\gamma + p_D D + p_m m}{p_\gamma + p_D D + p_m m + \theta V}.$$

The equilibrium price must ensure consistency between the market clearing condition and the first order condition of the optimization program of the active investors, requiring

$$\begin{aligned} \frac{\mu - rP + D}{r\gamma\sigma^2} - \frac{g'(V)\theta V}{r\gamma P} &= \frac{P - \theta V}{P}, \\ \mu - rP + D &= \frac{P - \theta V(1 - \frac{g'(V)}{r\gamma})}{P} r\gamma\sigma^2. \end{aligned} \quad (26)$$

In the constrained equilibrium, $\theta = 0$, and the Sharpe ratio equals to the supply of bonds normalized to 1 (see [Veronesi, 1999](#)). In general, however, a portion of investors may exert price pressure unrelated to fundamentals. In order for active investors to be comfortable with the equilibrium, the Sharpe ratio must decrease as price pressure increases. Let us workout the left-hand-side of the Equation (26).

$$\begin{aligned} \mu - rP + D &= \frac{\frac{m}{r} + \theta[rV(1 - \theta) + QD]}{1 - \theta Q} - rP + D \\ &= \frac{P}{P - \theta^2 V} \left(\frac{m}{r} + \theta[rV(1 - \theta) + QD] \right) - rP + D \\ &= \frac{1}{1 - \theta Q} \left(\frac{m}{r} + \theta[rV(1 - \theta) + QD] - r(P - \theta^2 V) \right) + D \\ &= \frac{1}{1 - \theta Q} \left(\frac{m}{r} + \theta[rV(1 - \theta) + QD] - rp_\gamma - D - \frac{m}{r} - r\theta V(1 - \theta) \right) + D \\ &= \frac{1}{1 - \theta Q} \left(D(\theta Q - 1) - rp_\gamma \right) + D = \frac{1}{1 - \theta Q} \left(D \frac{\theta^2 V - P}{P} - rp_\gamma \right) + D \\ &= -\frac{P}{P - \theta^2 V} rp_\gamma, \end{aligned} \quad (27)$$

Which uses the equivalence $\frac{1}{1 - \theta Q} = \frac{P}{P - \theta^2 V}$. Turning to the right-hand-side of the Equation (26),

$$\begin{aligned}
\frac{P - \theta V \left(1 - \frac{g'(V)}{r\gamma}\right)}{P} r\gamma\sigma^2 &= \frac{P - \theta V \left(1 - \frac{g'(V)}{r\gamma}\right)}{P} \left(\frac{\omega}{r} + \theta\pi\right)^2 \\
&= \frac{P - \theta V \left(1 - \frac{g'(V)}{r\gamma}\right)}{P} \left(\frac{\omega}{r} + \theta\pi\right)^2 \\
&= \frac{P - \theta V \left(1 - \frac{g'(V)}{r\gamma}\right)}{P} \left(\frac{\omega}{r} + \theta\pi\right)^2 \left(\frac{P}{P - \theta^2 V}\right)^2 \\
&= \frac{P - \theta^2 V}{P} \left(\frac{\omega}{r} + \theta\pi\right)^2 \left(\frac{P}{P - \theta^2 V}\right)^2 \\
&= \left(\frac{P}{P - \theta^2 V}\right) \left(\frac{\omega}{r} + \theta\pi\right)^2.
\end{aligned}$$

Therefore, the requisite that the FOC and the market clearing condition simultaneously hold necessitates $1 - \frac{g'(V)}{r\gamma} = \theta$, satisfied when $g'(V) = (1 - \theta)r\gamma$. As a result, Equation (26) simplifies to

$$p_\gamma = -\frac{1}{r} \left(\frac{\omega}{r} + \theta\pi\right)^2.$$

In the constrained equilibrium, $\theta = 0$, and the required compensation for risk accounts for uncertainty over earnings. In the more comprehensive equilibrium with both active and passive investors, the required compensation for risk incorporates flow risk. Let us replace the FOCs into the HJB.

$$\begin{aligned}
0 &= \frac{1}{J} U(c^*) + \frac{1}{J} \frac{\mathbb{E}_t[dJ]}{dt} = r + \frac{1}{J} \frac{\mathbb{E}_t[dJ]}{dt} \\
&= r - \delta - r\gamma \left[\frac{1}{\gamma} (\log r - g(V) - \beta) + X^*(\mu - rP + D) \right] - g'(V) [rV(1 - \theta) + Q(\mu + D)] \\
&\quad + \frac{1}{2} (r\gamma\sigma X^*)^2 + \frac{1}{2} (g'(V)^2 - g''(V))(Q^2\sigma^2 + \pi^2) + r\gamma\sigma^2 g'(V) X^* Q.
\end{aligned}$$

Substituting X^* ,

$$\begin{aligned}
0 &= r - \delta - r\gamma \left[\frac{1}{\gamma} (\log r - g(V) - \beta) + \left(\frac{\mu - rP + D}{r\gamma\sigma^2} - \frac{g'(V)}{r\gamma} Q \right) (\mu - rP + D) \right] \\
&\quad - g'(V) [rV(1 - \theta) + Q(\mu + D)] + \frac{1}{2} \left[r\gamma\sigma \left(\frac{\mu - rP + D}{r\gamma\sigma^2} - \frac{g'(V)}{r\gamma} Q \right) \right]^2 \\
&\quad + \frac{1}{2} (g'(V)^2 - g''(V))(Q^2\sigma^2 + \pi^2) + r\gamma\sigma^2 g'(V) \left(\frac{\mu - rP + D}{r\gamma\sigma^2} - \frac{g'(V)}{r\gamma} Q \right) Q.
\end{aligned}$$

Simplifying the expression yields

$$\begin{aligned}
0 &= r - \delta - r(\log r - g(V) - \beta) - \frac{(\mu - rP + D)^2}{2\sigma^2} \\
&\quad - g'(V) [rV(1 - \theta) + Q(\mu + D) - Q(\mu - rP + D) + \frac{\pi^2}{2}] \\
&= r - \delta - r(\log r - g(V) - \beta) - \frac{(\mu - rP + D)^2}{2\sigma^2} - g'(V) [rV(1 - \theta) + rQP + \frac{\pi^2}{2}].
\end{aligned}$$

Equivalently,

$$\begin{aligned}
0 &= r - \delta - r(\log r - g(V) - \beta) - \frac{(\mu - rP + D)^2}{2\sigma^2} - g'(V) [rV + \frac{\pi^2}{2}] \\
&= r - \delta - r(\log r - g(V) - \beta) - \frac{1}{2} \left(\frac{\omega}{r} + \theta\pi \right)^2 - g'(V) [rV + \frac{\pi^2}{2}].
\end{aligned}$$

We have used the equivalence $(\mu - rP + D)^2/\sigma^2 = (\frac{\omega}{r} + \theta\pi)^2$ from the Equations (24) and (27). We further know $g(V) = (1 - \theta)r\gamma V + K$, thus $g'(V) = (1 - \theta)r\gamma$, and $g''(V) = 0$. After replacing $\beta = \frac{(\gamma\omega)^2}{2r} + \frac{\delta}{r} + \log(r) - 1$,

$$\begin{aligned}
0 &= r - \delta - r(\log r - g(V) - \beta) - \frac{1}{2} \left(\frac{\omega}{r} + \theta\pi \right)^2 - g'(V) [rV + \frac{\pi^2}{2}] \\
&= \frac{(\gamma\omega)^2}{2} - rK - \frac{1}{2} \left(\frac{\omega}{r} + \theta\pi \right)^2 - (1 - \theta)r\gamma \frac{\pi^2}{2}.
\end{aligned}$$

It is immediate to see that the guess satisfies the requisite optimality and market clearing conditions for suitable constant K . The transversality condition is respected. From Equation (25) and the investors' wealth dynamics,

$$\lim_{h \rightarrow \infty} \mathbb{E} \left[J_{t+h} \right] = \lim_{h \rightarrow \infty} \mathbb{E} \left[-e^{-\delta(t+h) - r\gamma W_{t+h} - r\gamma(1-\theta)V_{t+h} - \beta} \right] = 0.$$

The equilibrium of Proposition 1 achieves as a special case when the asset allocation mandate of passive investors θ equals to 0, restraining them from allocating their wealth into equity markets.

Q.E.D.

B Proof of Proposition 3

It is insightful to inspect the following benchmark equilibria.

1) First, consider the equilibrium with *only active investors*, by setting $\theta = 0$. By the optimality of active investors, the price of the i -th stock takes the following standard form

$$P_{it} = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} D_{is} ds \right] = p_g + p_D D_{it} + p_m m_i,$$

where we recall that $p_g = -\frac{\gamma\omega^2}{r^2}$ and $\omega = \sqrt{\sum_i \omega_i^2 + \sum_i \sum_j \varrho_{ij}}$.

2) Second, consider the equilibrium with *only asset allocation investors*, that achieves with $r \rightarrow \infty$, alluring active investors to the riskless asset, and $\lambda_i = 0$, excluding the stock from the index. By market clearing, the price of the i -th stock is

$$P_{it} = q_{it}\theta V_t^A.$$

3) Third, consider the equilibrium with *asset allocation investors and index trackers*, that achieves with $r \rightarrow \infty$, alluring active investors to the riskless asset, and $\lambda_i = 1$, for stocks included in the index. By market clearing, the price of the i -th stock is

$$P_{it} = q_{it}\theta V_t^A + \lambda_i \theta V_t^{IDX}.$$

4) In general, market participants are *active investors, asset allocation investors, and index trackers*. Motivated by the benchmark equilibria above, guess that the equilibrium price of the i -th stock takes the following form.

$$P_{it} = p_\gamma + p_D D_{it} + p_m m_{it} + \theta [q_{it} V_t^A + \lambda_i V_t^{IDX}],$$

which results in individual stock price drift, volatility, and pairwise correlations given by

$$\mu_{it} = \frac{\frac{m_i}{r} + \theta[rV_t(1-\theta) + Q_{it}D_{it}]}{1 - \theta Q_{it}}, \quad \sigma_{it} = \frac{\frac{\omega_i}{r} + \theta\pi}{1 - \theta Q_{it}}, \quad \sigma_{ijt} = \frac{\frac{\varrho_{ij}}{r^2} + (\theta\pi)^2}{(1 - \theta Q_{it})(1 - \theta Q_{jt})}.$$

Asset allocation investors solve their mean-variance portfolio selection problem with FOCs

$$q_{it} = \frac{\mu_{it} - rP_{it} + D_{it}}{\gamma\sigma_{it}^2} - \frac{\sum_{j \neq i} q_{jt}\sigma_{ijt}}{\sigma_{it}^2},$$

$$\sum_{i \in I} q_{it} = 1.$$

The portfolio weights of index trackers are $\lambda_i = 1/N$. Respectively, asset allocation investors' wealth V_t^A and index trackers' wealth V_t^{IDX} follow dynamics

$$dV_t^A = rV_t^A(1 - \theta)dt + \theta V_t^A \sum_{i \in I} q_{it} \frac{dP_{it} + (D_{it} - rP_{it})dt}{P_{it}} + \pi^A dF_t,$$

$$dV_t^{IDX} = rV_t^{IDX}(1 - \theta)dt + \theta V_t^{IDX} \sum_{i \in I} \lambda_i \frac{dP_{it} + (D_{it} - rP_{it})dt}{P_{it}} + \pi^{IDX} dF_t.$$

The sum of the wealth of asset allocation investors and index trackers delivers the passive wealth tracking the stock market $V_t = V_t^A + V_t^{IDX}$, which follows Equation (4)

$$dV_t = rV_t(1 - \theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t.$$

This aggregation greatly simplifies the problem of the active investors, who can simply keep track of aggregate passive wealth V – that is, the state variable which influences changes in the investment opportunity set over time.

The HJB equation of active investors is

$$0 = \max_{\{c, X\}} U(c) + J_t + J_W \mathbb{E}_t[dW] + J_V \mathbb{E}_t[dV] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW^2] + \frac{1}{2} \mathbb{E}_t[dV^2] + J_{WV} \mathbb{E}_t[dW dV],$$

where

$$dW = (rW - c)dt + \sum_{i \in I} X_i [(D_i - rP_i)dt + dP_i].$$

Moreover,

$$\begin{aligned}
\mathbb{E}_t[dW] &= [rW - c + \sum_{i \in I} X_i(\mu_i - rP_i + D_i)]dt, \\
\mathbb{E}_t[dW^2] &= \mathbb{E}_t \left[\left(\sum_{i \in I} X_i dP_i \right)^2 \right] = \left(\sum_{i \in I} X_i \sigma_i \right)^2 dt + \left(\sum_{i \neq j} X_i X_j \sigma_{ij} \right) dt, \\
\mathbb{E}_t[dV] &= [rV(1 - \theta) + \sum_{i \in I} Q_i(\mu_i + D_i)]dt, \\
\mathbb{E}_t[dV^2] &= \left(\sum_{i \in I} Q_i \sigma_i \right)^2 dt + \left(\sum_{i \neq j} Q_i Q_j \sigma_{ij} \right) dt + \pi^2 dt, \\
\mathbb{E}_t[dVdW] &= \left(\sum_{i \in I} X_i Q_i \sigma_i^2 \right) dt + \left(\sum_{i \neq j} X_i Q_j \sigma_{ij} \right).
\end{aligned}$$

The value function is again $J(W, V, t) = -e^{-\delta t - r\gamma W - g(V) - \beta}$, and the FOCs of the HJB are

$$\begin{aligned}
X_{it}(P_i, D_i, V) &= \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma_{it}^2} - \frac{\sum_{j \neq i} X_{jt}\sigma_{ijt}}{\sigma_{it}^2} - \frac{g'(V)}{r\gamma} Q_{it}, \\
c_t(W, V) &= rW_t + \frac{1}{\gamma}(g(V) + \beta - \log r).
\end{aligned}$$

The remainder of the proof verifies the guess by following [Merton \(1973\)](#) and the steps outlined in [Appendix A](#).

C Extensions and Generalizations

C.1 Flows and Past Performance

When flows are correlated with past performance, passive investors' wealth dynamics is

$$dV = [rV(1 - \theta) + \pi\theta\alpha_t]dt + Q(dP + Ddt) + \pi dF,$$

where the expected flow α_t depend on the entire history of past earnings surprises,

$$\alpha_t = \int_{-\infty}^t e^{-\kappa(t-s)} dB_s.$$

Therefore, the expected flow increases in earnings news and reverts to its mean at rate κ

$$d\alpha_t = -\kappa\alpha_t dt + dB_t.$$

The equilibrium stock price in rational financial markets continues to satisfy

$$P_t = \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} D_s ds \right]}_{\text{Fundamentals}} + \underbrace{\mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} \theta V_{t+dt} \right]}_{\text{Wealth Allocated}}.$$

Fundamentals are unaffected, but the present discounted value of the wealth allocated to the equity market becomes

$$\mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} \theta V_{t+dt} \right] = \theta V_t + \theta \mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} \alpha_{t+dt} \right]$$

Define the expected flows given the recent history of earnings surprises as

$$\mathcal{I}_t = \underbrace{\theta \mathbb{E}_t^{\mathbb{Q}} \left[e^{-rdt} \alpha_{t+dt} \right]}_{\text{Conditional Flows}}. \quad (28)$$

The stock price is given by

$$P_t = PDV_t(D) + \theta V_t + \mathcal{I}_t. \quad (29)$$

In fact, wealth invested θV_t grows at the risk-free rate in risk-adjusted terms, and \mathcal{I}_t is the present discounted value of flows conditional on the history of earnings.

Using Itô's Lemma,

$$dP = p_D dD + \theta [rV(1 - \theta)dt + Q(dP + Ddt) + \pi dF] + \alpha_t \theta dt + d\mathcal{I}_t.$$

The amplification of earnings becomes stronger by a factor which corresponds to the predictive power of earnings surprises for the stream of passive flows. Intuitively, the rational forecast of wealth pressure compares the previous known history of earnings surprises to the future ones, which are instead unpredictable. Price pressure is stronger as negative surprises move back in time. In conclusion, the wealth flows grow with past earnings at rate $\theta\alpha_t dt$. Expected flows $d\mathcal{I}_t$ are higher after good news, inducing persistence in the time series of returns.

C.2 The Treasury Market

In the context of the model derived in this paper, asset allocation strategies require passive investors to invest a fixed proportion of their wealth $(1 - \theta)V_t$ in the bond market, which thus exerts price pressure on the Treasury price. A general equilibrium perspective would complicate the problem of active investors without affecting the main intuitions. However, a partial equilibrium specification for the bond market is insightful. The main difference from [Greenwood and Vayanos \(2014\)](#) is that the price pressure V_t represents here a demand factor rather than a supply factor. The maintained assumption on the bond market is that active investors have agile demand which does not separate prices from fundamentals, while passive investors do not attempt to time the bond market. The contribution is to model the demand for bonds resulting from wealth effects on the stock market.

The structure in [Greenwood and Vayanos \(2014\)](#) suggests that a shock to the demand factor should move the yields of all bonds in the opposite direction as the shock. Moreover, a shock to the demand factor should the instantaneous expected returns of all bonds in the opposite direction as the shock. After an exogenous increase to the wealth passively invested, one should thus observe lower yields and bond expected returns. [Hartzmark and Solomon \(2022\)](#) document that days in the top quintile of dividend payments are associated with higher market returns. The amount of dividends is determined ahead of the dividend pay date, and hence the effect documented cannot be ascribed to information. The impact of dividend price pressure has increased since 1990, as passive mutual funds and ETFs have become a larger component of equity holdings. Dividend payout days are thus of interest for the assessment of wealth effects. [Figure A.3](#) provides evidence suggestive of the spillover of wealth effects between the U.S. equity and Treasury bond market using dividend pay dates as a clean instrument for wealth shocks unrelated to information. The pattern revealed by the data is clear and sizeable. Days with large dividend payment amounts feature large returns on the stock market (Panel A), with low term premia (Panel B), and low expected returns on 10-year Treasury bonds (Panel C).

D Relation with Previous Contributions

[Basak and Pavlova \(2013\)](#) consider an economy populated by retail investors and institutional agents who care about their performance relative to an index. In the effort to outperform their benchmark, institutional agents generate upward price pressure on the

stocks included in the index. Additionally, institutional investors increase the volatility of returns and amplify the effect of cash flow news. In the cross section, institutional investors induce excess correlation among stocks included in an index without affecting comparable assets outside the index.

The framework proposed in this paper differs in many respects. First, the present paper is interested in the demand for the equity asset class in the form of asset allocation mandates, rather than the demand for assets in the benchmark within the equity asset class. For example, asset allocation mutual funds have the explicit mandate of dividing their investments among different asset classes, such as stocks and bonds, regardless of changes in the investment opportunity set. Conversely, active investors such as HFs and sophisticated households are more agile and during market downturns often fly to the safety offered by bonds. As a result, investors with fixed allocation mandates generate an asset class effect associated with the entire equity market, which goes over and above the known index inclusion effect. The entire stock market reflects the wealth allocated to the asset class independently of the fluctuations of its Sharpe ratio, and generates “cash-in-the-market” pricing effects. These effects extend to the cross section, where the correlation of stocks in excess of their common fundamentals depends not only by the inclusion in the index and the wealth tracking the index, but also varies as a function of the wealth allocated to the stock market. For example, the paper finds that the wealth invested into asset allocation and equity mutual funds affects the correlations between non-index stocks.

Second, the focus on the equity asset class yields implication for the growth rate of the entire stock market. The wealth passively allocated to the stock market raises the expected price changes and reduces expected returns, consistently with the empirical evidence. Thus, this paper offers some guidance on the effects of passive investing on the first moment of returns and price changes. Moreover, wealth effects on the stock market depend by the prevailing market conditions on the bond market, and vice versa. Importantly, the level of stock prices, as well as portfolio holdings, amplification effects, and correlations, depend analytically by the wealth dynamically invested by the groups of agents considered, rather than from the static proportion of each group. More technical differences include the infinite horizon of the model. Moreover, dividends are distributed continuously rather than at a terminal date. Nonetheless, the framework is very tractable.

Third, the derived model offers implications for the informativeness of stock prices about the future behavior of earnings. In [Basak and Pavlova \(2013\)](#), the relative share of retail and institutional investors is a parameter used to perform comparative statics.

In this paper, the proportion of passive investors is dynamic and stochastic wealth flows into asset prices. As a reflection of wealth flows into investment vehicles with fixed asset allocation mandates, markets are incomplete, since demand risk is unspanned and prices may change without news about the future behavior of fundamentals. The magnitude of these effects varies endogenously, and depends on the strength of earnings reported by companies. Specifically, price efficiency rises with the incentive of active investors to invest in stocks, as captured by their Sharpe ratio, and decreases with the position of passive investors.

Gabaix and Koijen (2022) propose a framework to analyze the fluctuations in the aggregate stock market where households allocate capital to institutions who are constrained in their equity share. As a result, flows in and out of the stock market have large impact of prices. Proposition 2 in their paper models the fractional change of the aggregate demand for equity q of the fund with asset allocation mandate θ as follows.

$$q = -\zeta p + \kappa \delta d + f, \quad \zeta = 1 - \theta + \kappa \delta.$$

In the above, ζ is the price elasticity of demand, and p is the fractional price change from the baseline. Moreover, κ is a measure of flexibility of the mixed fund, δ is the baseline price/dividend ratio, d is the proportional variation of the expected dividend from the baseline value, and f is the equity-holdings weighted fractional flow. In **Gabaix and Koijen (2022)**, investment mandates amplify the effects of flows since the macro elasticity $\zeta < 1$ and the demand multiplier, the inverse of the macro elasticity, is greater than one. Specifically, $\zeta = 0.16$ in Table 6. Indeed, the authors analyze what happens after equity inflows $f > 0$, maintaining constant the dividend growth $d = 0$. As the supply of shares does not change, we must have $q = 0$ in the equilibrium after the news. We thus have $0 = -\zeta p + f$, and therefore

$$p = \frac{f}{\zeta} > f.$$

Consider however the asset pricing effects of good news about fundamentals $d > 0$, maintaining constant flows $f = 0$. As the supply of shares does not change, one must have $q = 0$ in the equilibrium after the news. Thus, $0 = -\zeta p + \kappa \delta d$, and hence

$$p = \frac{\kappa \delta}{\zeta} d = \frac{\kappa \delta}{1 - \theta + \kappa \delta} d < d,$$

as long as the asset allocation mandate $\theta < 1$. Specifically, $\theta = 0.875$ in Table 5. Therefore, passive investors *mitigate* the amplification of fundamentals. This feature appears at variance with the data shown in Figure 2, where movements of fundamentals are amplified by the rise in passive investing. Moreover, the amplification is constant while in the present paper it dynamically depends on the shares held by passive investors.

The fundamental distinctions are thus that [Gabaix and Kojien \(2022\)](#) focus on the market elasticity, that is, the response of prices to demand shocks. By contrast, the central amplification mechanism of the present paper is on the response of prices to innovations, including both economic fundamentals and demand shocks. Furthermore, this paper shows that asset prices reflect the *stock* of wealth passively invested on the stock market rather than stochastic capital *flows*. As a result, the amplification effect and the resulting market volatility derived in this paper are dynamic rather than static. Finally, thanks to the generality of its structure, the setup derived comfortably generates interesting implications for the cross section of stocks.

E Data Description

The event study around earnings announcements is conducted using daily stock information from CRSP, quarterly “street” earning reports from the actuals I/B/E/S files, balance sheet variables from COMPUSTAT quarterly, and benchmarking intensity data from [Pavlova and Sikorskaya \(2023\)](#).

Filters are standard and require CRSP ordinary stocks (share code 10 and 11) from the daily security file to trade on NYSE, AMEX, or Nasdaq (exchange codes 1, 2, and 3). The three pricing models considered to construct normal returns are the benchmark stock-level constant mean, the CAPM, and the Fama-French 3 factor model. The advantage of the former is to minimize estimation noise. The latter two specifications are estimated on a rolling window of 1 year and lagged by 1 quarter from the event date. The residuals of these models are the abnormal returns. The paper uses quarterly earnings per share (EPS) from I/B/E/S to construct standardized unexpected earnings (SUE), measured as the increment in EPS over four quarters divided by their rolling standard deviation estimated over 8 quarters. Earnings reported on weekends or on weekdays after 16:00 Eastern Time are imputed to the first date on which is possible to trade on the information. Benchmarking intensity is recorded at the yearly frequency and at the stock level every June from 1998 to 2018. The variable is defined as the cumulative weight of a stock across benchmarks scaled by the amount of assets following each benchmark and

divided by the market capitalization of the stock, and thus directly maps to $Q = \theta V/P$.

The data are then merged. To reduce the influence of outliers in a large sample, each quarter the SUE observations above and below three standard deviations from the mean are dropped. To alleviate the effects of microcaps and estimation noise, every year the observations below the 5th percentile of market value are dropped, as in [Jegadeesh and Titman \(2001\)](#), and abnormal returns are winsorized at the 1st and 99th percentiles. The final sample is composed of 5,568 firms for the constant mean model, which does not require rolling estimates, and 5,516 firms for the CAPM and FF3 models, the estimates of which require 250 valid trading days per company. The sample offers thus a good representation of the universe of U.S. stocks during the past two decades.

TABLE 1: **Parameter Estimates of GARCH-MIDAS with Passive Holdings** The Table presents parameter estimates of the component model relating volatility realized on day d to its lags and a long-run component of the proportion of the U.S. stock market held by MFs and ETFs in quarter q . The data are from Bloomberg and the Flow of Funds, both variables are expressed in percentage terms, and numbers in parentheses are robust t-statistics.

PANEL A: FIXED LONG-RUN COMPONENT

Sample	m	a	b	c	w ₁	w ₂	n	LLF/BIC
1951-2019	0.06494 (11.472)	0.08811 (42.374)	0.90279 (341.23)	0.02398 (4.709)	47.224 (0.0020)	2.111 (0.0009)	0.77061 (9.0894)	-19247.1 38562.5
1953-1984	0.06036 (7.9467)	0.08365 (18.027)	0.90796 (178.36)	0.10584 (1.6083)	1.0014 (0.0179)	49.84 (0.0019)	0.33247 (1.5682)	-7446.29 14955.5
1953-2010	0.06407 (10.485)	0.08052 (41.639)	0.91165 (358.17)	0.05200 (4.8109)	48.918 (0.0875)	49.51 (0.0862)	0.60517 (7.345)	-16429.4 32925.9
1985-2010	0.06273 (5.4763)	0.06210 (15.001)	0.92964 (197.23)	0.01406 (1.961)	37.334 (0.0224)	49.773 (0.0227)	0.91895 (5.8952)	-7506.17 15073.9

PANEL B: ROLLING LONG-RUN COMPONENT

Sample	m	a	b	c	w ₁	w ₂	n	LLF/BIC
1951-2019	0.06495 (11.498)	0.08819 (42.348)	0.90267 (340.77)	0.02437 (4.5952)	44.66 (0.0326)	17.662 (0.0330)	0.77657 (9.4833)	-19247.3 38562.8
1953-1984	0.06034 (7.9491)	0.08370 (17.882)	0.90791 (176.58)	0.10727 (1.6381)	1.7811 (0.0384)	49.884 (0.0352)	0.3278 (1.5766)	-7446.23 14955.4
1953-2010	0.06404 (10.486)	0.08046 (41.221)	0.91171 (358.3)	0.05280 (4.5349)	40.498 (0.0757)	49.881 (0.0777)	0.60621 (7.2433)	-16429.4 32925.9
1985-2010	0.06273 (5.4728)	0.06209 (15.025)	0.92964 (198)	0.01487 (1.9869)	37.947 (0.0214)	49.754 (0.0217)	0.9076 (5.5916)	-7506.1 15073.7

The specification is:

$$\begin{aligned}
 r_{d,q} &= m + \sqrt{l_q} g_{d,q} e_{d,q}, \\
 l_q &= n + c \sum_{k=1}^K f_k(w_1, w_2) \text{Passive Share}_{q-k}, \\
 g_{d,q} &= (1 - a - b) + a \frac{(r_{d-1,q} - m)^2}{l_q} + b g_{d-1,q}.
 \end{aligned}$$

The beta weighting function f_k has $K = 16$ lags, $r_{d,q}$ is the S&P 500 return. The innovation $e_{d,q}$ is white noise, Passive Share_q is the proportion of the stock market held by mutual funds and exchange-traded funds, and the remaining terms are parameters.

TABLE 2: **Descriptive Statistics** The Table presents summary statistics of the sample of corporations used in the event study. Balance sheet variables are from Compustat. Earnings per Share is from I/B/E/S, and Standardized Unexpected Earnings are computed as the yearly difference in Earnings per Share divided by their eight-quarters trailing volatility. Benchmarking Intensity is the measure of wealth tracking each stock proposed by [Pavlova and Sikorskaya \(2023\)](#). The sample runs from 1998 to 2018.

Variable	Observations	Mean	Median	Std. Dev.	Skewness
Total Assets	126,803	12710.37	1583.92	83497.32	19.39
Total Liabilities	126,768	9998.765	890.85	74809.12	19.79
Earnings per Share	125,124	1.498261	1.2490	3.826793	14.88
Benchmarking Intensity	135,582	0.179089	0.1894	0.079325	-0.55
Standardized Unexpected Earnings	135,582	0.289230	0.2515	1.686929	-0.23

TABLE 3: Event Study around Earnings Announcements The Table presents the result of a regression of daily abnormal returns and cumulative abnormal returns on standardized earnings surprises and its interaction with the wealth passively tracking the stock. A unit of observation is an announcement of earnings of a U.S. company reported between 1998 and 2018. The numbers in parentheses are robust standard errors.

PANEL A: ABNORMAL RETURNS						
	Constant Mean		CAPM		FF3	
	(1)	(2)	(3)	(4)	(5)	(6)
Earnings Surprise _{it}	0.276*** (0.034)	0.280*** (0.034)	0.280*** (0.035)	0.285*** (0.036)	0.281*** (0.036)	0.287*** (0.036)
Earnings Surprise _{it} × Wealth Benchmarked _{it}	0.577*** (0.174)	0.575*** (0.175)	0.514*** (0.182)	0.513*** (0.182)	0.510*** (0.183)	0.507*** (0.183)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Year	Quarter	Year	Quarter	Year	Quarter
No. Obs.	135,577	135,577	124,468	124,468	124,468	124,468
\bar{R}^2	0.010	0.013	0.012	0.014	0.012	0.014
Firms	5,568	5,568	5,516	5,516	5,516	5,516

PANEL B: CAR(-1, 1)						
	Constant Mean		CAPM		FF3	
	(1)	(2)	(3)	(4)	(5)	(6)
Earnings Surprise _{it}	0.334*** (0.037)	0.335*** (0.038)	0.336*** (0.038)	0.338*** (0.038)	0.335*** (0.039)	0.339*** (0.039)
Earnings Surprise _{it} × Wealth Benchmarked _{it}	0.472** (0.188)	0.481** (0.188)	0.423** (0.194)	0.434** (0.194)	0.429** (0.195)	0.438** (0.195)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Year	Quarter	Year	Quarter	Year	Quarter
No. Obs.	135,577	135,577	124,468	124,468	124,468	124,468
\bar{R}^2	0.011	0.019	0.018	0.023	0.018	0.021
Firms	5,568	5,568	5,516	5,516	5,516	5,516

PANEL C: CAR(-3, 3)						
	Constant Mean		CAPM		FF3	
	(1)	(2)	(3)	(4)	(5)	(6)
Earnings Surprise _{it}	0.374*** (0.042)	0.380*** (0.043)	0.374*** (0.043)	0.378*** (0.043)	0.385*** (0.044)	0.391*** (0.044)
Earnings Surprise _{it} × Wealth Benchmarked _{it}	0.479** (0.211)	0.475** (0.210)	0.445** (0.219)	0.458** (0.219)	0.397* (0.221)	0.407* (0.221)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Year	Quarter	Year	Quarter	Year	Quarter
No. Obs.	135,577	135,577	124,468	124,468	124,468	124,468
\bar{R}^2	0.012	0.030	0.034	0.043	0.033	0.040
Firms	5,568	5,568	5,516	5,516	5,516	5,516

Internet Appendix for “Dynamic Asset Pricing with Passive Investing”

Ruggero Jappelli

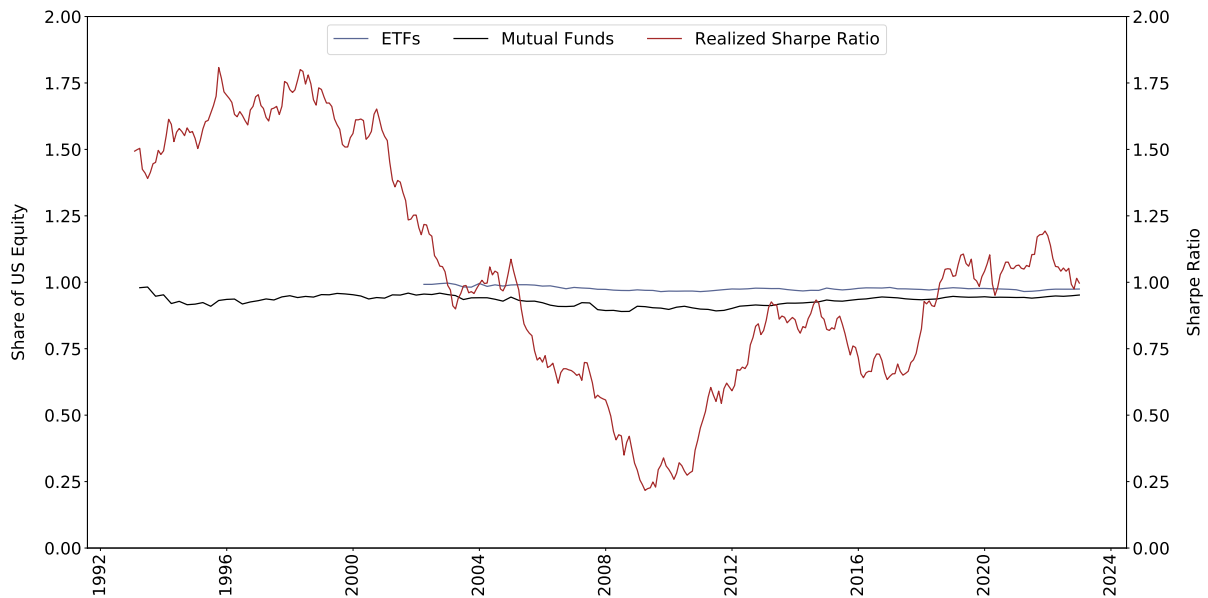


FIGURE A.1: **Funds’ Asset Allocation and the Investment Opportunity Set.** The figure shows the average share of U.S. Equity held by Mutual Funds and ETFs using monthly data from Morningstar (left y-axis) and the monthly realized Sharpe ratio using data from CRSP and the from Kenneth French data library (right y-axis). The sample includes the universe of funds classified as U.S. Equity, Sector Equity, Allocation, and International Equity. The U.S. Equity shares are aggregated with weights corresponding to the assets under management of the fund. The realized Sharpe ratio of the U.S. Equity asset class is computed as the monthly return on the value-weighted CRSP index in excess of the risk-free rate, divided by the one-year rolling volatility of returns.

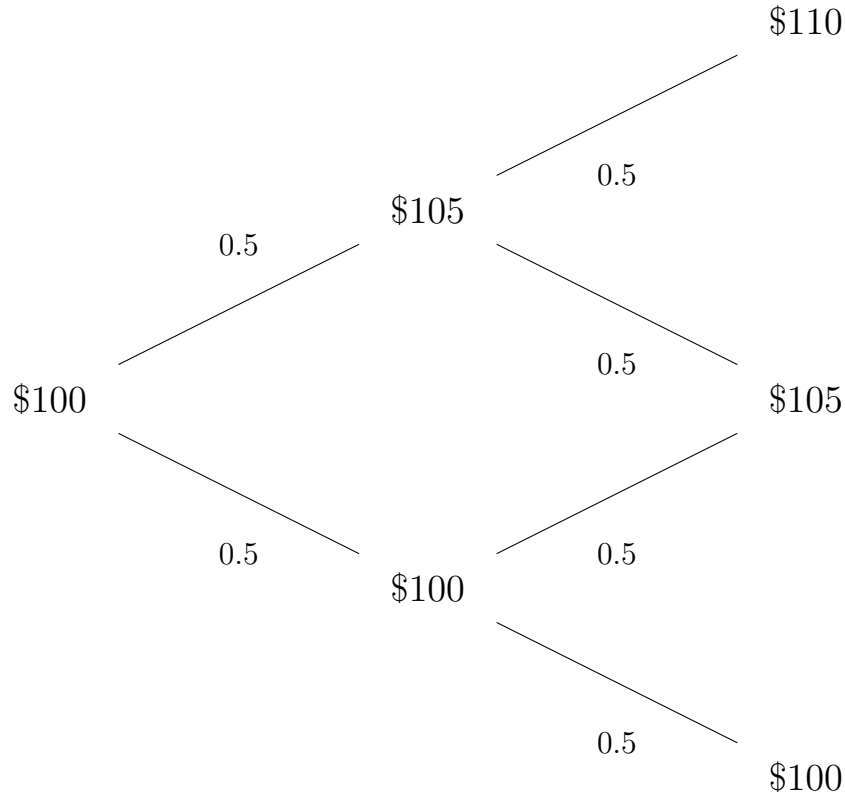


FIGURE A.2: **Path Dependency of Passive Investments.** For illustrative purposes, suppose the price exogenously follows the above specified binomial tree, and set $\gamma = 10$, and $a = r = 0$ to dividends and opportunity costs. The economy starts at time $t = 0$, runs for two periods, and the price goes either *Up* or *Down*. At each node, $\mu = 2.5$ and $\sigma^2 = 0.25$, thus $\mu - rP = \gamma\sigma^2$. The time $t = 1$ demand of active investors is independent of the past, $X_1(Up) = X_1(Down) = 1$. However, the time $t = 1$ demand of passive investors has memory of the past, and if $\theta \leq 1$ then $Q_1(Up) \leq Q_1(Down)$. For instance, if $V_0 = 200\$$ and $\theta = 0.5$, $Q_1(Up) = 0.976$ while $Q_1(Down) = 1$. The level of wealth invested by passive investors is procyclical, since $V_1(Up) = 102.5$ and $V_1(Down) = 100$. Hence, the decision of passive investors is path dependent. In this illustration, prices are exogenous. Wealth amplification effects achieve when the magnitude of the passive investments V_t affects the price of the risky asset.

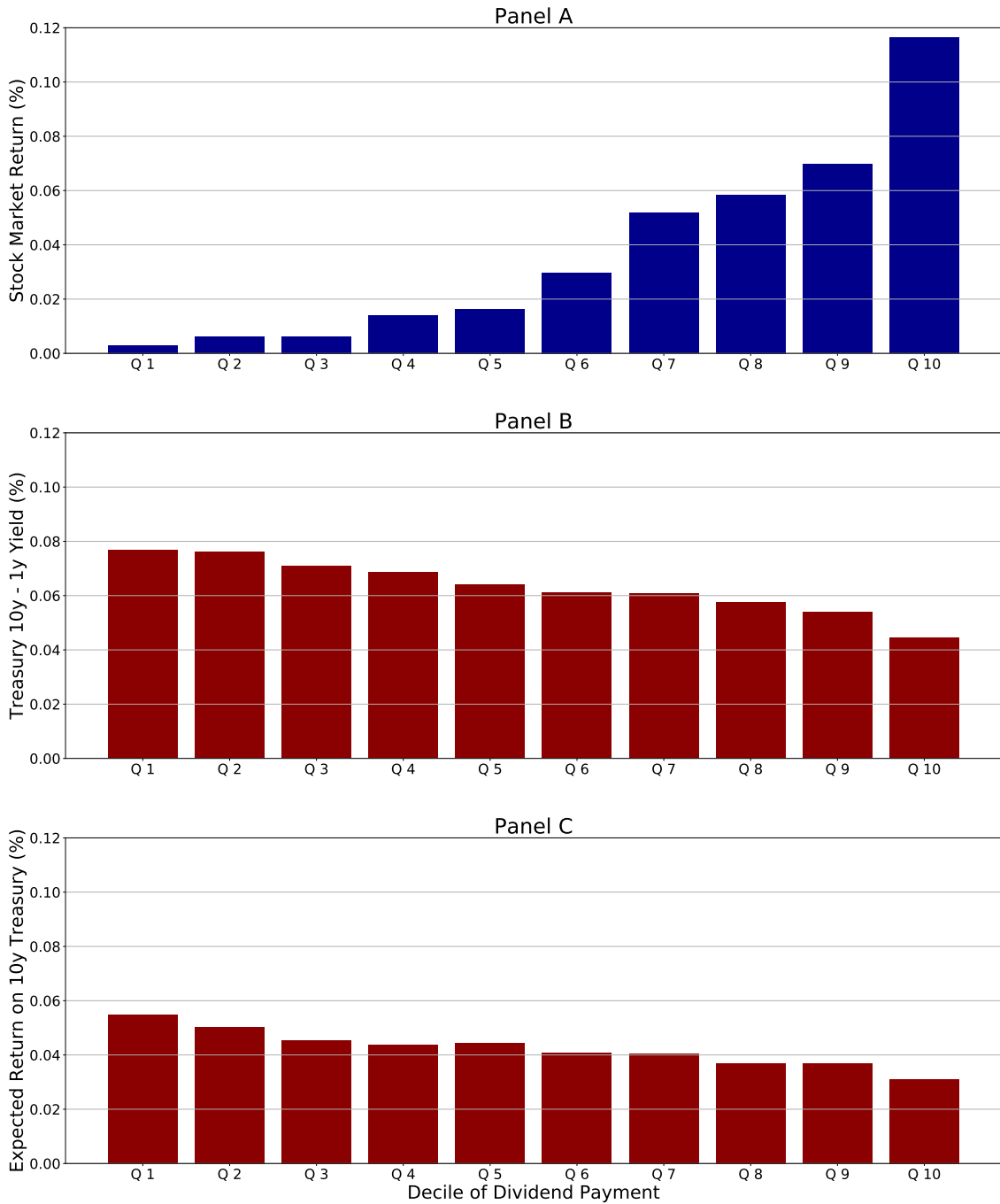


FIGURE A.3: **Wealth Effects in Equity and Treasury Markets.** Trading days are grouped into deciles by dividend payment amount, which are reported on the x-axis. Panel A: the y-axis shows the return on the value-weighted stock market index averaged within each decile. Panel B: the y-axis shows the return on the 10-year U.S. Treasury in excess of the 1-year U.S. Treasury averaged within each decile. Panel C: the y-axis shows the expected return on the 10-year U.S. Treasury averaged within each decile. Daily data from CRSP and GSW.

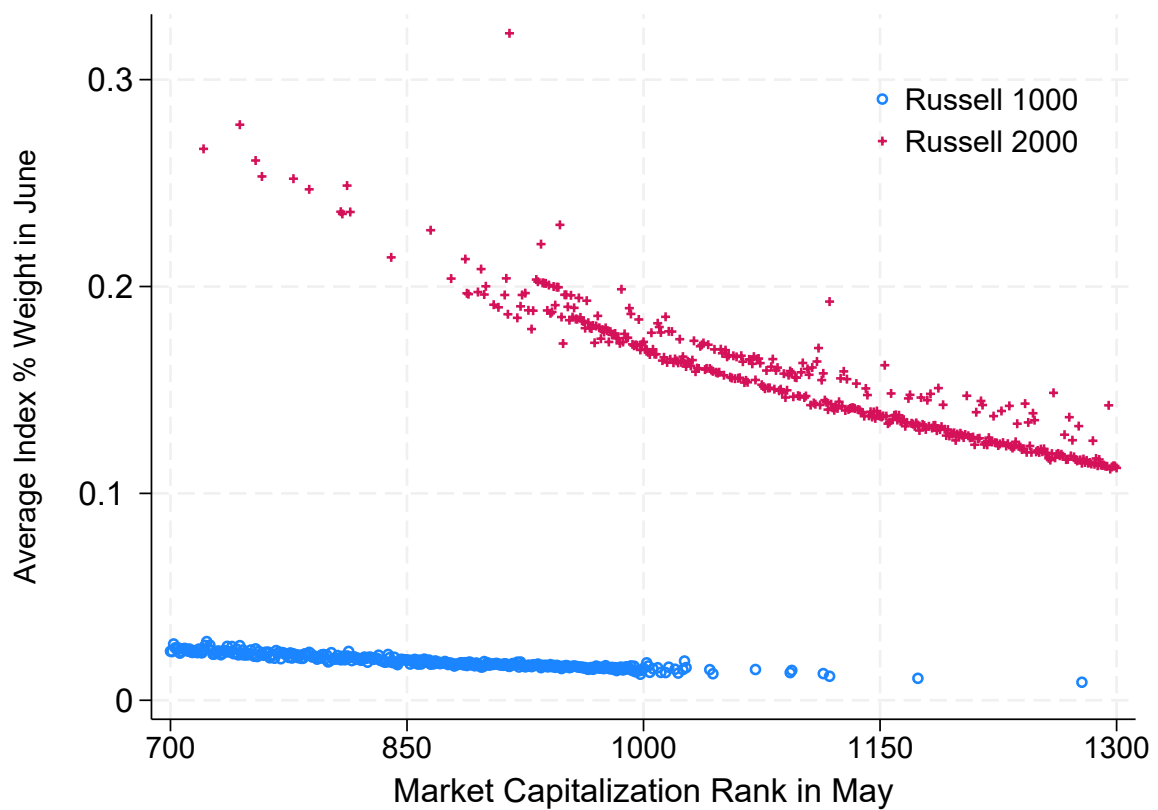


FIGURE A.4: **Discontinuity in the Index Weights around the Russell Cutoff.** The figure plots the time average of the index weight in June, computed as the float-adjusted market capitalization of each stock relative to that of its index, against the float-adjusted market capitalization rank in May. Data from 1996 to 2006 are obtained from Bloomberg and CRSP.

References

- Adam, K., A. Marcet, and J. P. Nicolini. 2016. Stock market volatility and learning. *The Journal of Finance* 71:33–82.
- Allen, F., and D. Gale. 2005. From cash-in-the-market pricing to financial fragility. *Journal of the European Economic Association* 3:535–546.
- Baele, L., G. Bekaert, and K. Inghelbrecht. 2010. The determinants of stock and bond return comovements. *The Review of Financial Studies* 23:2374–2428.
- Bai, J., T. Philippon, and A. Savov. 2016. Have financial markets become more informative? *Journal of Financial Economics* 122:625–654.
- Barbon, A., and V. Gianinazzi. 2019. Quantitative easing and equity prices: Evidence from the ETF program of the Bank of Japan. *The Review of Asset Pricing Studies* 9:210–255.
- Basak, S., and A. Pavlova. 2013. Asset prices and institutional investors. *American Economic Review* 103:1728–1758.
- Ben-David, I., F. Franzoni, and R. Moussawi. 2018. Do ETFs increase volatility? *The Journal of Finance* 73:2471–2535.
- Berkman, H., and P. D. Koch. 2017. DRIPs and the dividend pay date effect. *Journal of Financial and Quantitative Analysis* 52:1765–1795.
- Betermier, S., L. E. Calvet, and E. Jo. 2023. A supply and demand approach to capital markets. Working Paper.
- Billio, M., M. Getmansky, A. W. Lo, and L. Pelizzon. 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of financial economics* 104:535–559.
- Black, F. 1976. Studies of stock market volatility changes. *Proceedings of the American Statistical Association, Business & Economic Statistics Section, 1976* .
- Bond, P., A. Edmans, and I. Goldstein. 2012. The real effects of financial markets. *Annual Review of Financial Economics* 4:339–360.
- Bond, P., and D. Garcia. 2022. The equilibrium consequences of indexing. *The Review of Financial Studies* 35:3175–3230.
- Brunnermeier, M. K., S. A. Merkel, and Y. Sannikov. 2022. Debt as safe asset. Working Paper.
- Brunnermeier, M. K., and Y. Sannikov. 2014. A macroeconomic model with a financial sector. *American Economic Review* 104:379–421.

- Buffa, A. M., D. Vayanos, and P. Woolley. 2022. Asset management contracts and equilibrium prices. *Journal of Political Economy* 130:3146–3201.
- Chaieb, I., H. Langlois, and O. Scaillet. 2021. Factors and risk premia in individual international stock returns. *Journal of Financial Economics* 141:669–692.
- Chang, Y.-C., H. Hong, and I. Liskovich. 2015. Regression discontinuity and the price effects of stock market indexing. *The Review of Financial Studies* 28:212–246.
- Chien, Y., H. Cole, and H. Lustig. 2011. A multiplier approach to understanding the macro implications of household finance. *The Review of Economic Studies* 78:199–234.
- Chien, Y., H. Cole, and H. Lustig. 2012. Is the volatility of the market price of risk due to intermittent portfolio rebalancing? *American Economic Review* 102:2859–2896.
- Coles, J. L., D. Heath, and M. C. Ringgenberg. 2022. On index investing. *Journal of Financial Economics* 145:665–683.
- Connolly, R., C. Stivers, and L. Sun. 2005. Stock market uncertainty and the stock-bond return relation. *Journal of Financial and Quantitative Analysis* 40:161–194.
- Coppola, A. 2021. In safe hands: The financial and real impact of investor composition over the credit cycle. Working Paper.
- Coval, J., and E. Stafford. 2007. Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics* 86:479–512.
- David, A., and P. Veronesi. 2013. What ties return volatilities to price valuations and fundamentals? *Journal of Political Economy* 121:682–746.
- Dávila, E., and C. Parlatore. 2023. Volatility and informativeness. *Journal of financial economics* 147:550–572.
- Duffee, G. R. 2023. Macroeconomic News and Stock–Bond Comovement. *Review of Finance* 27:1859–1882.
- Dumas, B., A. Kurshev, and R. Uppal. 2009. Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility. *The Journal of Finance* 64:579–629.
- D’Amico, S., and T. B. King. 2013. Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics* 108:425–448.
- Engle, R. F., E. Ghysels, and B. Sohn. 2013. Stock market volatility and macroeconomic fundamentals. *Review of Economics and Statistics* 95:776–797.
- Fama, E. F. 1970. Efficient capital markets: A review of theory and empirical work. *The Journal of Finance* 25:383–417.

- Fama, E. F., and K. R. French. 1992. The cross-section of expected stock returns. *the Journal of Finance* 47:427–465.
- Franzoni, F., and M. C. Schmalz. 2017. Fund flows and market states. *The Review of Financial Studies* 30:2621–2673.
- Gabaix, X., and R. S. Koijen. 2022. In search of the origins of financial fluctuations: The inelastic markets hypothesis. Working Paper.
- Gârleanu, N., and L. H. Pedersen. 2022. Active and passive investing: Understanding Samuelson’s dictum. *The Review of Asset Pricing Studies* 12:389–446.
- Ghysels, E., A. Plazzi, R. Valkanov, A. Rubia, and A. Dossani. 2019. Direct versus iterated multiperiod volatility forecasts. *Annual Review of Financial Economics* 11:173–195.
- Gourinchas, P.-O., W. Ray, and D. Vayanos. 2022. A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. Working Paper.
- Greenwood, R. 2008. Excess comovement of stock returns: Evidence from cross-sectional variation in Nikkei 225 weights. *The Review of Financial Studies* 21:1153–1186.
- Greenwood, R., and D. Vayanos. 2014. Bond supply and excess bond returns. *The Review of Financial Studies* 27:663–713.
- Grossman, S. J. 1995. Dynamic asset allocation and the informational efficiency of markets. *The Journal of Finance* 50:773–787.
- Grossman, S. J., and J. E. Stiglitz. 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70:393–408.
- Haddad, V., P. Huebner, and E. Loualiche. 2021. How competitive is the stock market? Theory, evidence from portfolios, and implications for the rise of passive investing. Working Paper.
- Haddad, V., and T. Muir. 2021. Do intermediaries matter for aggregate asset prices? *The Journal of Finance* 76:2719–2761.
- Harris, L., and E. Gurel. 1986. Price and volume effects associated with changes in the S&P 500 list: New evidence for the existence of price pressures. *The Journal of Finance* 41:815–829.
- Hartzmark, S. M., and D. H. Solomon. 2022. Predictable price pressure. Working Paper.
- Hasanhodzic, J., and A. W. Lo. 2019. On Black’s leverage effect in firms with no leverage. *The Journal of Portfolio Management* 46:106–122.

- He, Z., and A. Krishnamurthy. 2013. Intermediary asset pricing. *American Economic Review* 103:732–770.
- He, Z., and W. Xiong. 2013. Delegated asset management, investment mandates, and capital immobility. *Journal of Financial Economics* 107:239–258.
- Hodor, I., and F. Zapatero. 2023. Asset Pricing Implications of Heterogeneous Investment Horizons. Working Paper.
- Hotchkiss, E. S., and D. Strickland. 2003. Does shareholder composition matter? Evidence from the market reaction to corporate earnings announcements. *The Journal of Finance* 58:1469–1498.
- Hou, K., and M. A. Van Dijk. 2019. Resurrecting the size effect: Firm size, profitability shocks, and expected stock returns. *The Review of Financial Studies* 32:2850–2889.
- Jappelli, R., L. Pelizzon, and M. G. Subrahmanyam. 2023. Quantitative easing, the repo market, and the term structure of interest rates. Working Paper.
- Jegadeesh, N., and S. Titman. 2001. Profitability of momentum strategies: An evaluation of alternative explanations. *The Journal of finance* 56:699–720.
- Jermann, U. J., and V. Quadrini. 2007. Stock market boom and the productivity gains of the 1990s. *Journal of Monetary Economics* 54:413–432.
- Koijen, R. S., R. J. Richmond, and M. Yogo. 2020. Which investors matter for equity valuations and expected returns? Working Paper.
- Koijen, R. S., and M. Yogo. 2019. A demand system approach to asset pricing. *Journal of Political Economy* 127:1475–1515.
- Kraft, H., A. Meyer-Wehmann, and F. T. Seifried. 2020. Dynamic asset allocation with relative wealth concerns in incomplete markets. *Journal of Economic Dynamics and Control* 113:103857.
- Kyle, A. S., and W. Xiong. 2001. Contagion as a wealth effect. *The Journal of Finance* 56:1401–1440.
- Lettau, M., S. C. Ludvigson, and J. A. Wachter. 2007. The declining equity premium: What role does macroeconomic risk play? *The Review of Financial Studies* 21:1653–1687.
- Lettau, M., and S. Van Nieuwerburgh. 2008. Reconciling the return predictability evidence. *The Review of Financial Studies* 21:1607–1652.
- Lou, D. 2012. A flow-based explanation for return predictability. *The Review of Financial Studies* 25:3457–3489.

- Madhavan, A. 2003. The Russell reconstitution effect. *Financial Analysts Journal* 59:51–64.
- Maxted, P. 2023. A macro-finance model with sentiment. *Review of Economic Studies* p. 2674.
- Merton, R. C. 1973. An intertemporal capital asset pricing model. *Econometrica* pp. 867–887.
- Moskowitz, T. J., Y. H. Ooi, and L. H. Pedersen. 2012. Time series momentum. *Journal of Financial Economics* 104:228–250.
- Pandolfi, L., and T. Williams. 2019. Capital flows and sovereign debt markets: Evidence from index rebalancings. *Journal of Financial Economics* 132:384–403.
- Parker, J. A., A. Schoar, and Y. Sun. 2023. Retail financial innovation and stock market dynamics: The case of target date funds. *The Journal of Finance* 78:2673–2723.
- Pavlova, A., and T. Sikorskaya. 2023. Benchmarking intensity. *The Review of Financial Studies* 36:859–903.
- Sammon, M. 2022. Passive ownership and price informativeness. Working Paper.
- Scholes, M. S. 1972. The market for securities: Substitution versus price pressure and the effects of information on share prices. *The Journal of Business* 45:179–211.
- Sharpe, W. F. 1991. The arithmetic of active management. *Financial Analysts Journal* 47:7–9.
- Shiller, R. J., and A. E. Beltratti. 1992. Stock prices and bond yields: Can their comovements be explained in terms of present value models? *Journal of monetary economics* 30:25–46.
- Shleifer, A. 1986. Do demand curves for stocks slope down? *The Journal of Finance* 41:579–590.
- Stambaugh, R. F. 2014. Presidential address: Investment noise and trends. *The Journal of Finance* 69:1415–1453.
- Stein, J. C. 2009. Presidential address: Sophisticated investors and market efficiency. *The Journal of Finance* 64:1517–1548.
- Tang, K., and W. Xiong. 2012. Index investment and the financialization of commodities. *Financial Analysts Journal* 68:54–74.
- Tirole, J. 1982. On the possibility of speculation under rational expectations. *Econometrica* pp. 1163–1181.

- Vayanos, D., and J.-L. Vila. 2021. A preferred-habitat model of the term structure of interest rates. *Econometrica* 89:77–112.
- Vayanos, D., and P. Woolley. 2013. An institutional theory of momentum and reversal. *The Review of Financial Studies* 26:1087–1145.
- Veronesi, P. 1999. Stock market overreactions to bad news in good times: A rational expectations equilibrium model. *The Review of Financial Studies* 12:975–1007.
- Vissing-Jørgensen, A. 2002. Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy* 110:825–853.